USING ADVANCED COMPUTING IN APPLIED DYNAMICS: FROM THE DYNAMICS OF GRANULAR MATERIAL TO THE MOTION OF THE MARS ROVER

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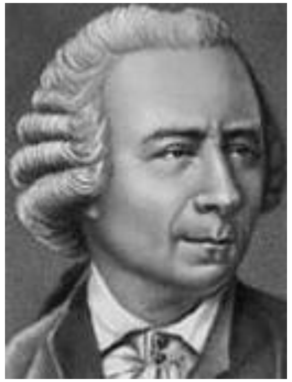
Classical Computational Multibody Dynamics: Newton-Euler Constrained Equations of Motion



Generalized Positions

Generalized Mass Matrix

Velocity Transformation Matrix

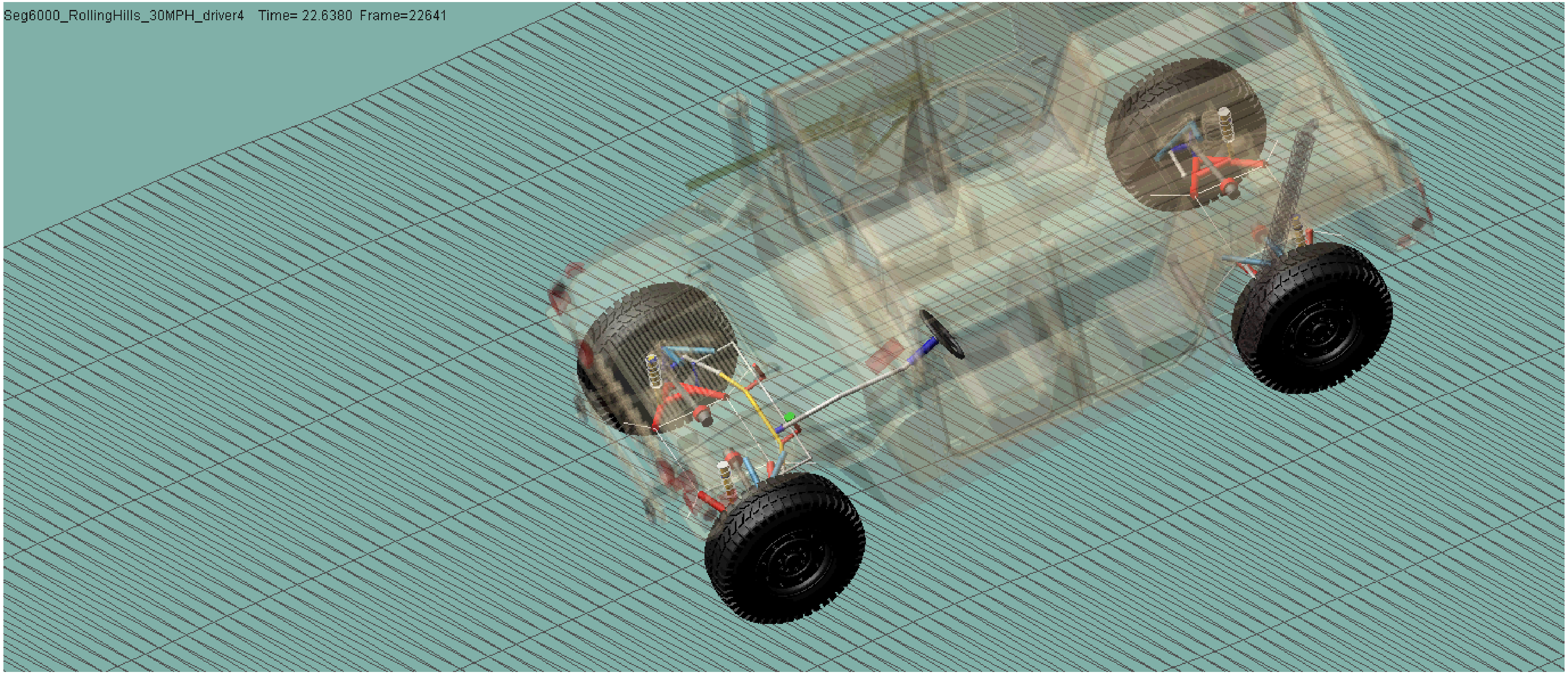
Generalized Velocities

Kinematic Differential Equations → $\dot{\mathbf{q}} = \mathbf{L}(\mathbf{q})\mathbf{v}$

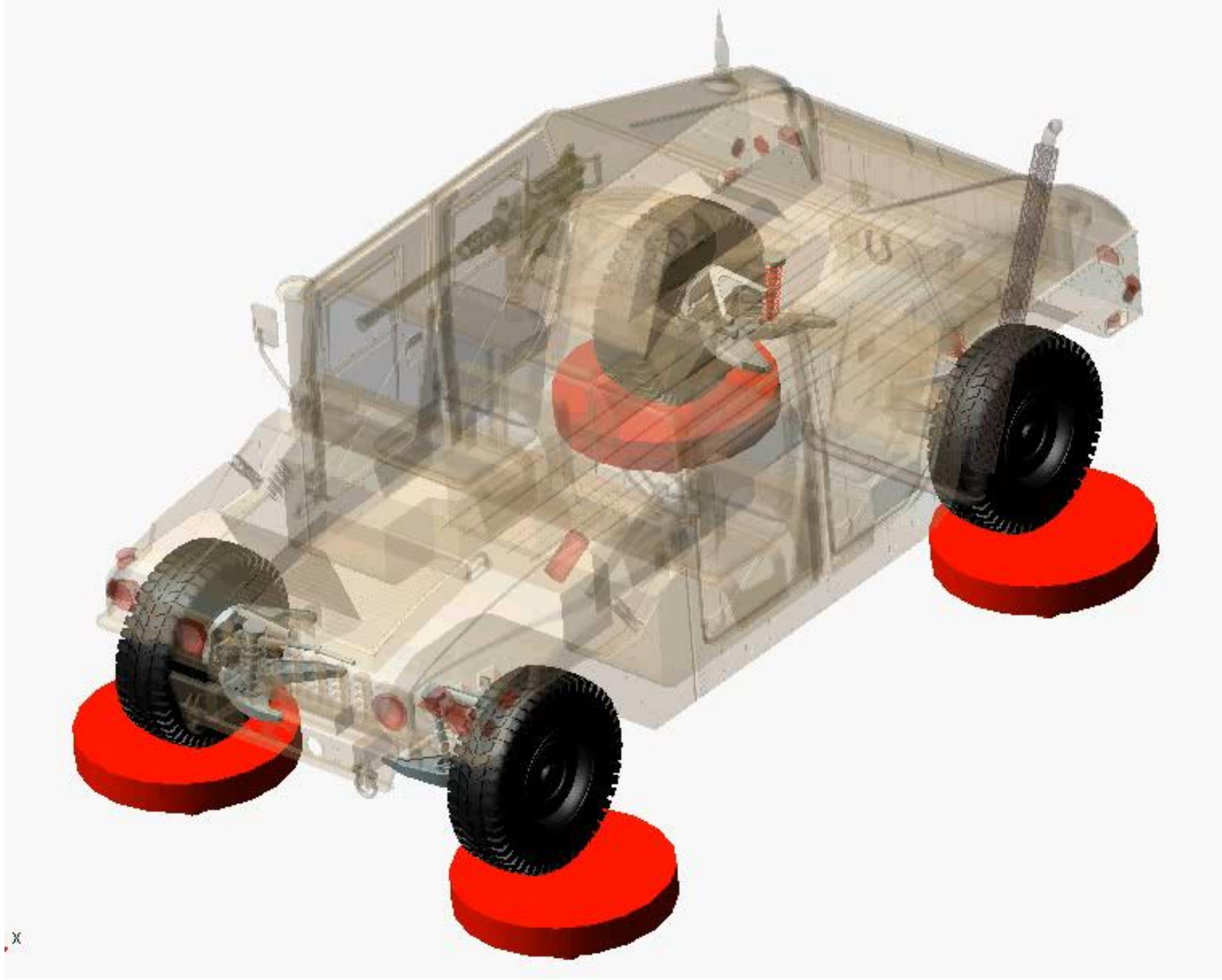
Force Balance Equations → $\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} = \underbrace{\mathbf{f}(t,\mathbf{q},\mathbf{v})}_{\text{Applied Force}} - \overbrace{\mathbf{g}_{\mathbf{q}}^{T}(\mathbf{q},t)\boldsymbol{\lambda}}^{\text{Reaction Force}}$

Holonomic Kinematic Constraints → $\mathbf{g}(\mathbf{q},t) = \mathbf{0}$

Multibody Dynamics: What is it? [commercial software simulation]



Multibody Dynamics: What is it? [commercial software simulation]

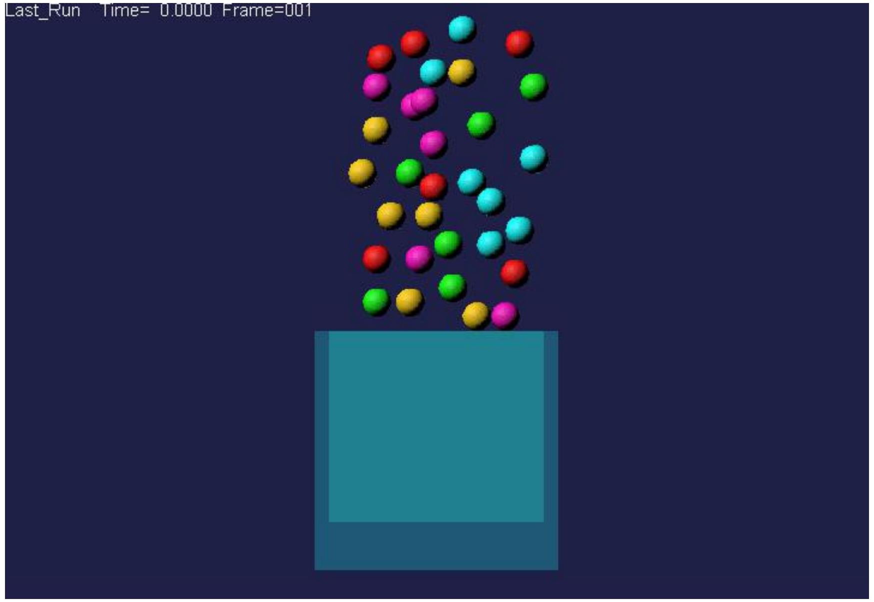


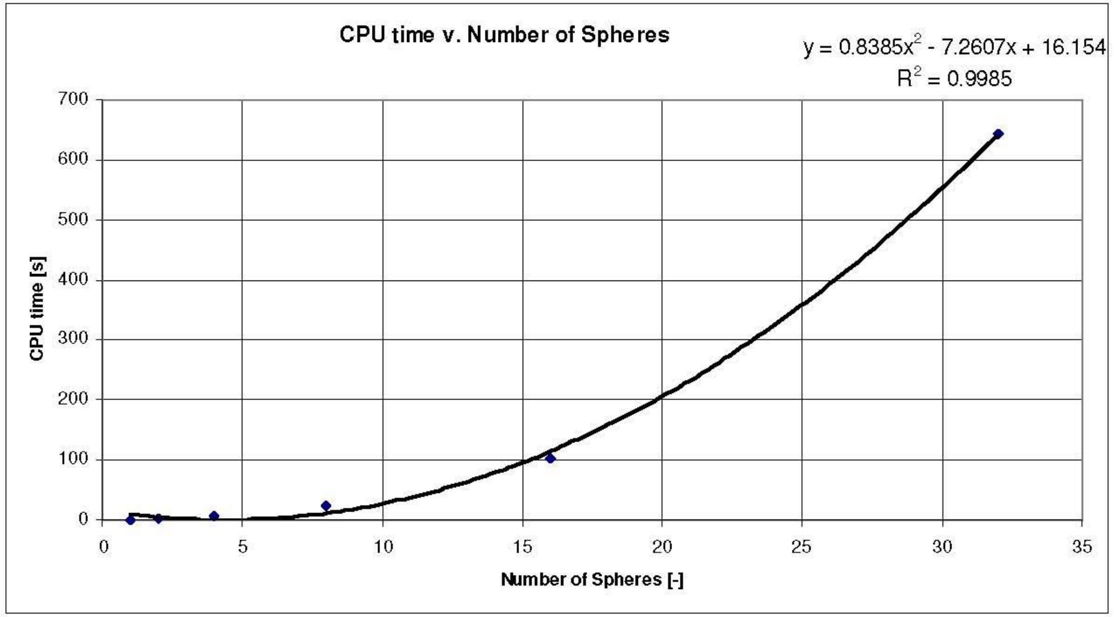
Example Open Problem: Mobility on Deformable Terrain

- How is the Rover moving along on a slope with granular material?
- What wheel geometry is more effective?
- How much power is needed to move it?
- At what grade will it get stuck?
- And so on…

Frictional Contact Simulation [Commercial Software Simulation - 2007]

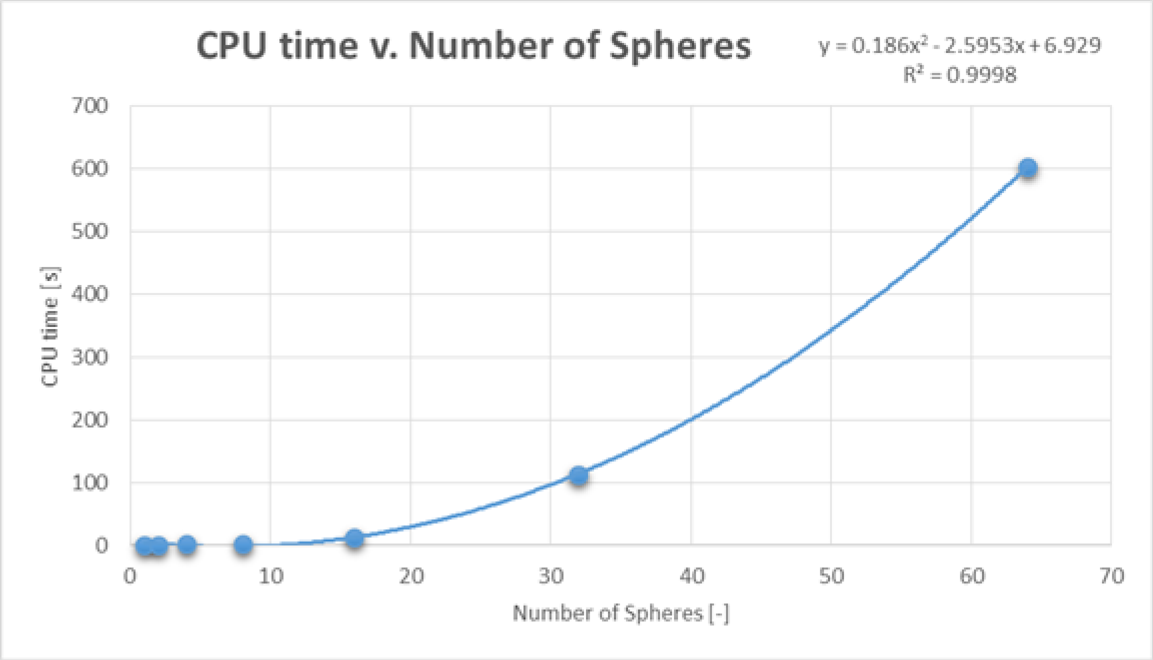
- Model Parameters:
  - Spheres: 60 mm diameter and mass 0.882 kg
  - Penalty Approach: stiffness of 1E5, force exponent of 2.2, damping coefficient of 10.0
  - Simulation length: 3 seconds



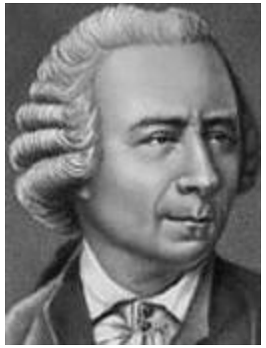


Frictional Contact Simulation [Commercial Software Simulation - 2013]

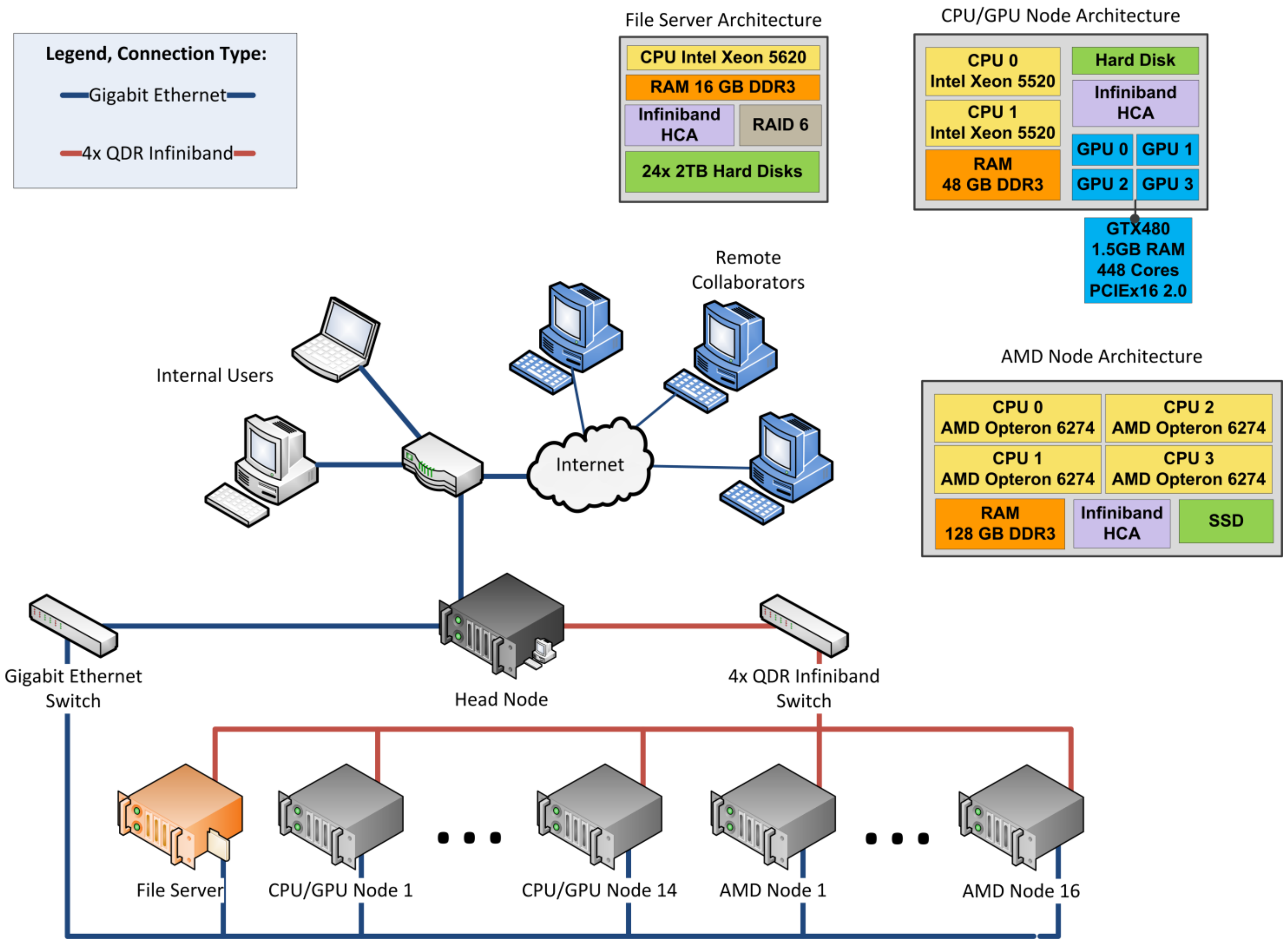
- Same problem tested in 2013
- Simulation time reduced by a factor of six
- Simulation times still prohibitively long



Why It's Worth Reconsidering Challenging Problems…



Lab's Research Heterogeneous Cluster



Lab's Research Heterogeneous Cluster

- More than 50,000 GPU scalar processors
- More than 1,200 CPU cores
- Fast Mellanox Infiniband Interconnect (QDR), 40Gb/sec
- About 2.7 TB of RAM
- More than 20 Tflops Double Precision

The issues is not hardware availability. Rather, it is producing modeling and solution techniques that can leverage the hardware

CHRONO:

Research-Grade Software Infrastructure for Multi-physics Modeling/Simulation/Visualization

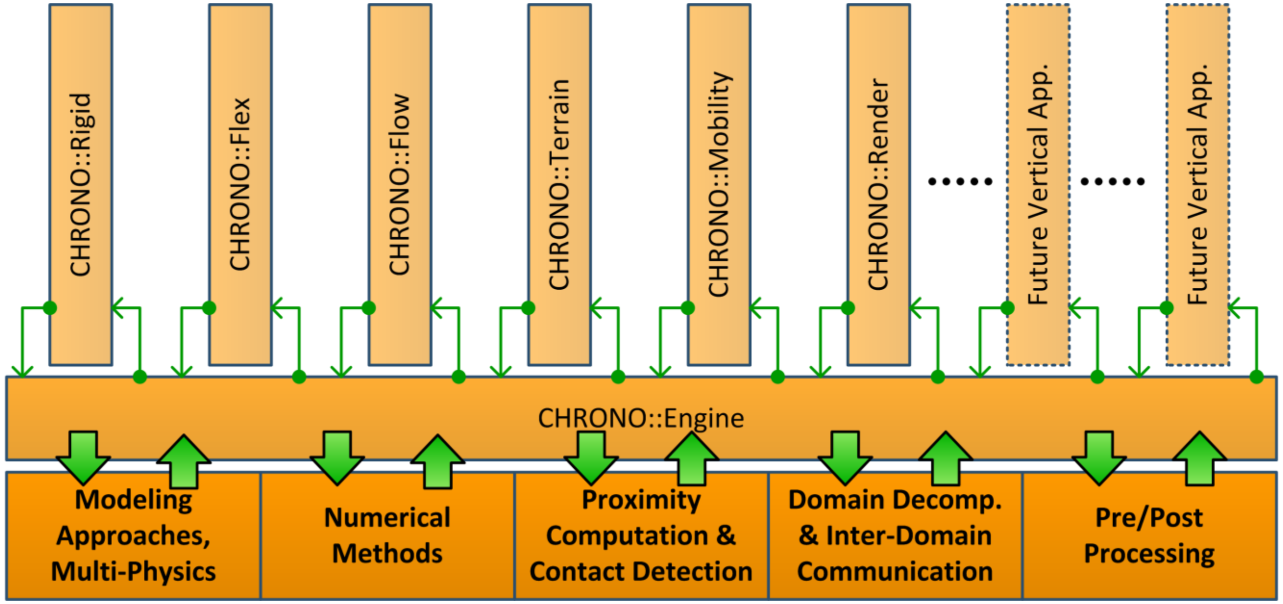
- Goal: advance state of the art in modeling, simulation, and visualization
  - Use emerging hardware and novel algorithms to solve open engineering problems

- "emerging hardware" :
  - GPUs and clusters of CPUs

- "open engineering problems" :
  - Fluid-solid interaction, vehicle mobility, soil modeling, tire/terrain modeling, granular dynamics, etc.

Chrono:
Five Foundation Components

- Advanced modeling
- Solution methods
- Proximity computation
- Domain decomposition & Inter-domain data exchange
- Pre/Post-processing (visualization)



- **Chrono**:
  - Five foundation components support vertical apps

Advanced Modeling Techniques

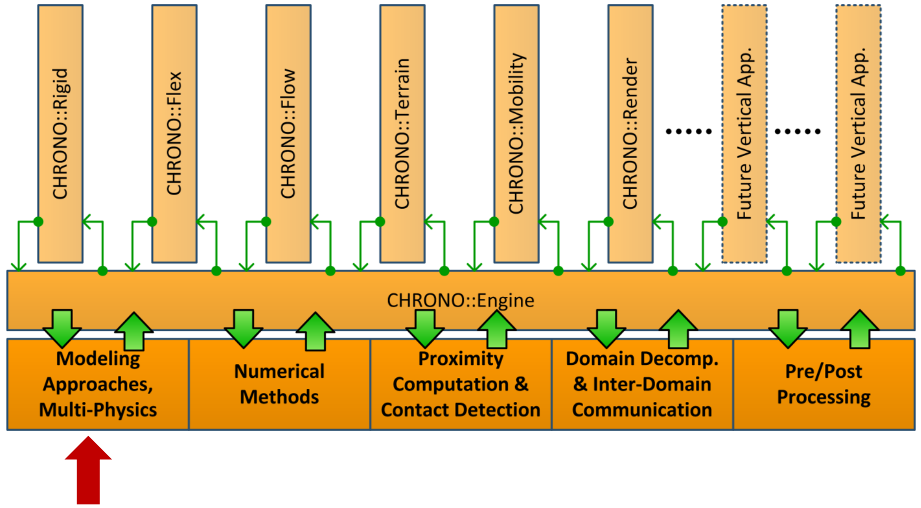
Advanced modeling techniques

Algorithmic (applied math) support

Proximity computation

Domain decomposition & Inter-domain data exchange

Post-processing (visualization)

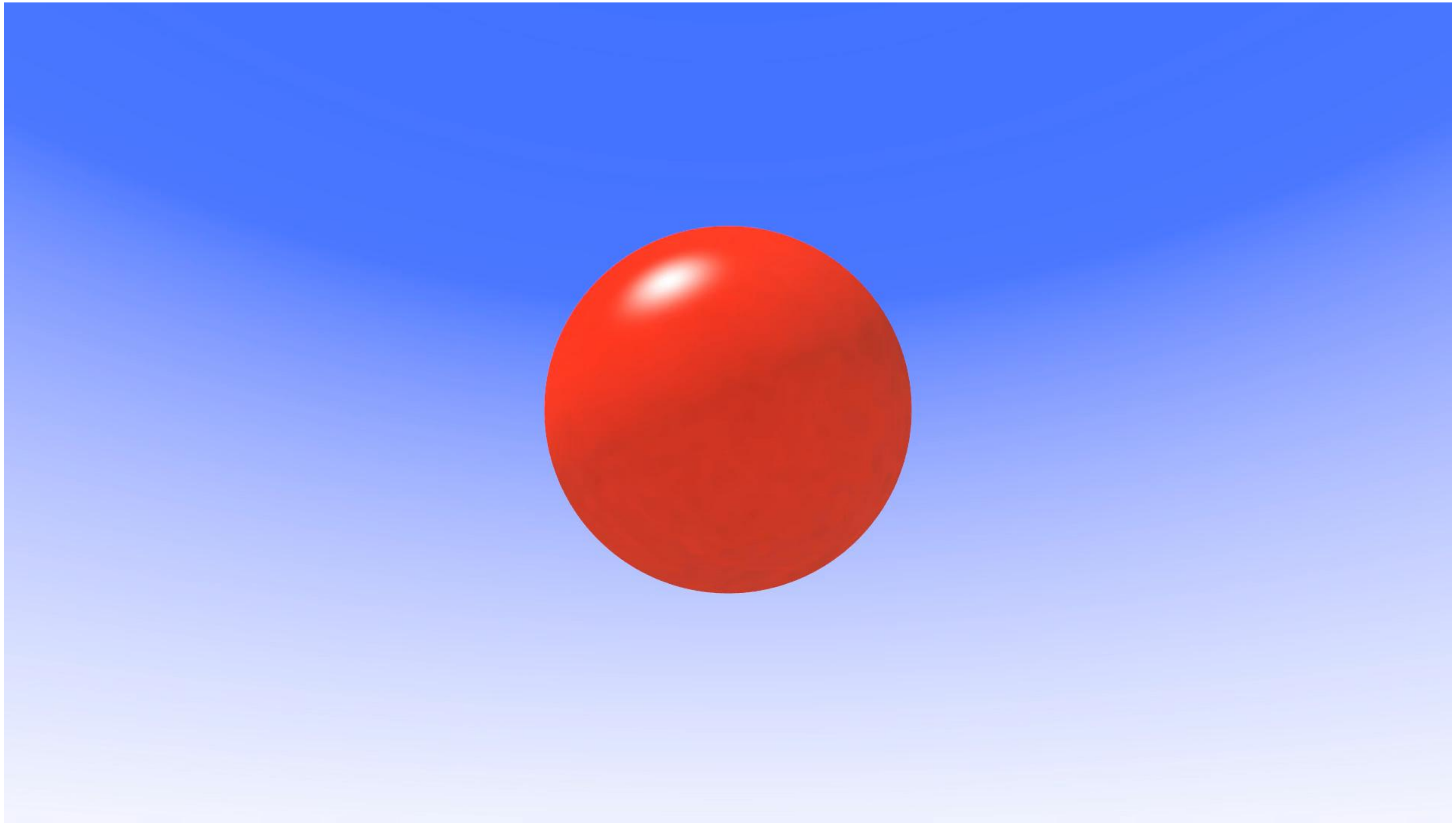


Chrono: Support for Advanced Modeling Techniques

- Modeling; what does it mean?
  - The process of formulating a set of governing differential equations that captures the physics associated with the engineering problem of interest

- Modeling decisions are consequential
  - Hallmark of good modeling: it leads to a palatable math problem that can be solved numerically with relative ease

Chrono::Flex - Dealing with Compliant Bodies



Deformable Body Modeling Support in **Chrono**

Equation of Motion & Mass Matrix

- Equations of Motion:

$$\mathbf{M}\ddot{\mathbf{e}} + \mathbf{Q}_s = \mathbf{Q}_e$$

- Mass matrix is constant and SPD

$$\mathbf{M} = \left[\int_{V_o} \rho \mathbf{S}^{\mathrm{T}} \mathbf{S} \, dV_o \right]$$

System Forces

- Due to gravity

$$\mathbf{Q}_e = A \int_0^l \mathbf{S}^T \mathbf{f}_g \, dx$$

- Due to a concentrated force:

$$\mathbf{Q}_e = \mathbf{S}^T \mathbf{f}$$

Deformable Bodies: Internal Forces…

- Strain Energy (shown for beam elements):

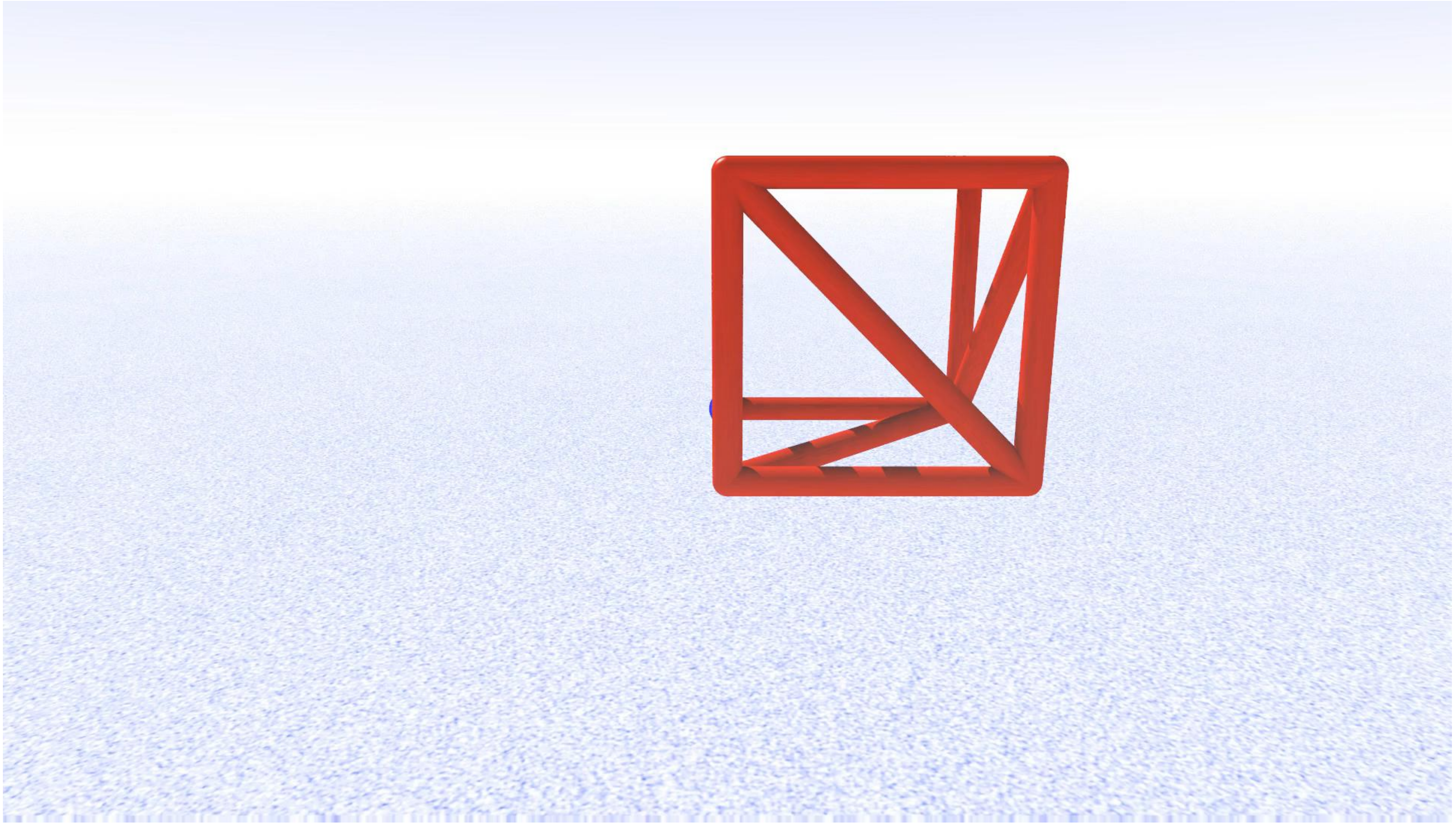
$$U = \frac{1}{2}\int_0^l EA(\varepsilon_{11})^2\,dx + \frac{1}{2}\int_0^l EI(\kappa)^2\,dx$$

- Partial Derivative of Strain Energy wrt generalized coordinated **e** yields the Internal Forces

$$\mathbf{Q}_s = \int_0^l EA(\varepsilon_{11})\left(\frac{\partial \varepsilon_{11}}{\partial \mathbf{e}}\right)^T dx + \int_0^l EI(\kappa)\left(\frac{\partial \kappa}{\partial \mathbf{e}}\right)^T dx$$

- Bad news: internal force expensive to evaluate
- Good News: for large systems can be done in parallel

Deformable Bodies with Constraints



Deformable Bodies with Constraints

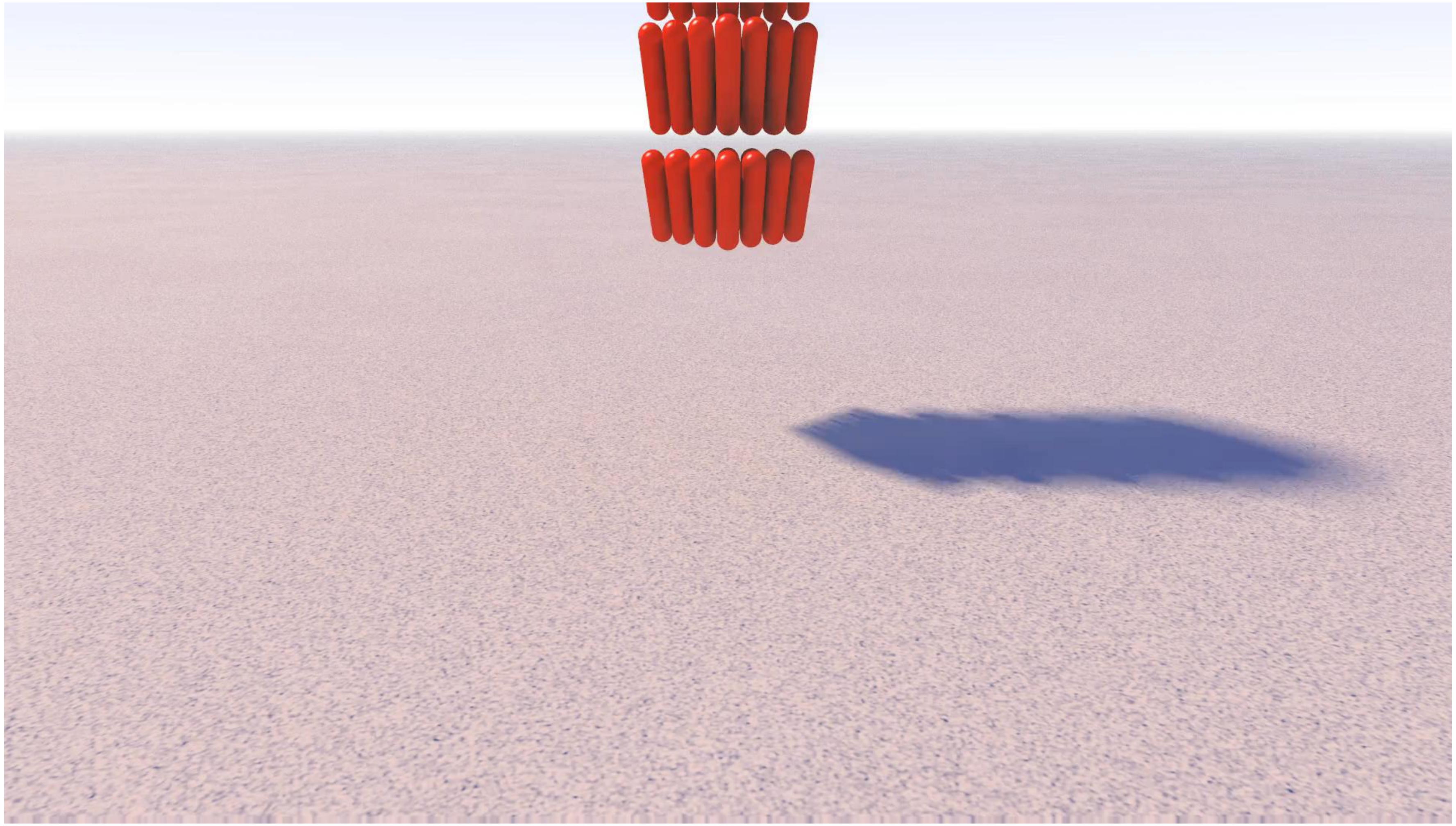
- Constraints assumed holonomic, formulated as

$$\mathbf{\Phi}(\mathbf{q},t) = [\Phi_1(\mathbf{q},t)\ldots\Phi_m(\mathbf{q},t)]^T = 0$$

- Constraint form of the Equations of Motion (index 3 DAE problem)

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{\Phi}_{\mathbf{q}}^T(\mathbf{q},t)\lambda + \mathbf{Q}_{\text{int}}(\mathbf{q}) = \mathbf{Q}_{\text{ext}}(\dot{\mathbf{q}},\mathbf{q},t)$$

Wiggly Bodies
[Flexible bodies, w/ Friction and Contact: parallel simulation on the GPU]

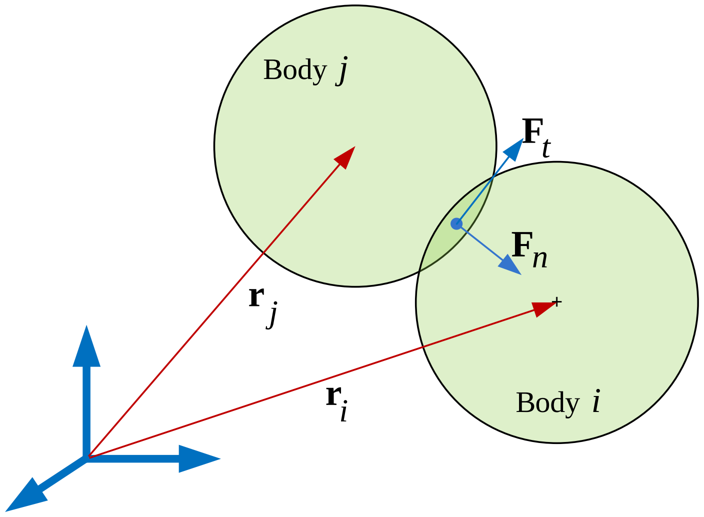


Deformable Bodies with Friction and Contact

- Contact forces depend on several parameters:
  - Contact penetration
  - Normal vector
  - Relative velocity of colliding bodies at the point of contact
  - Etc.
- Normal force due to a collision calculated as

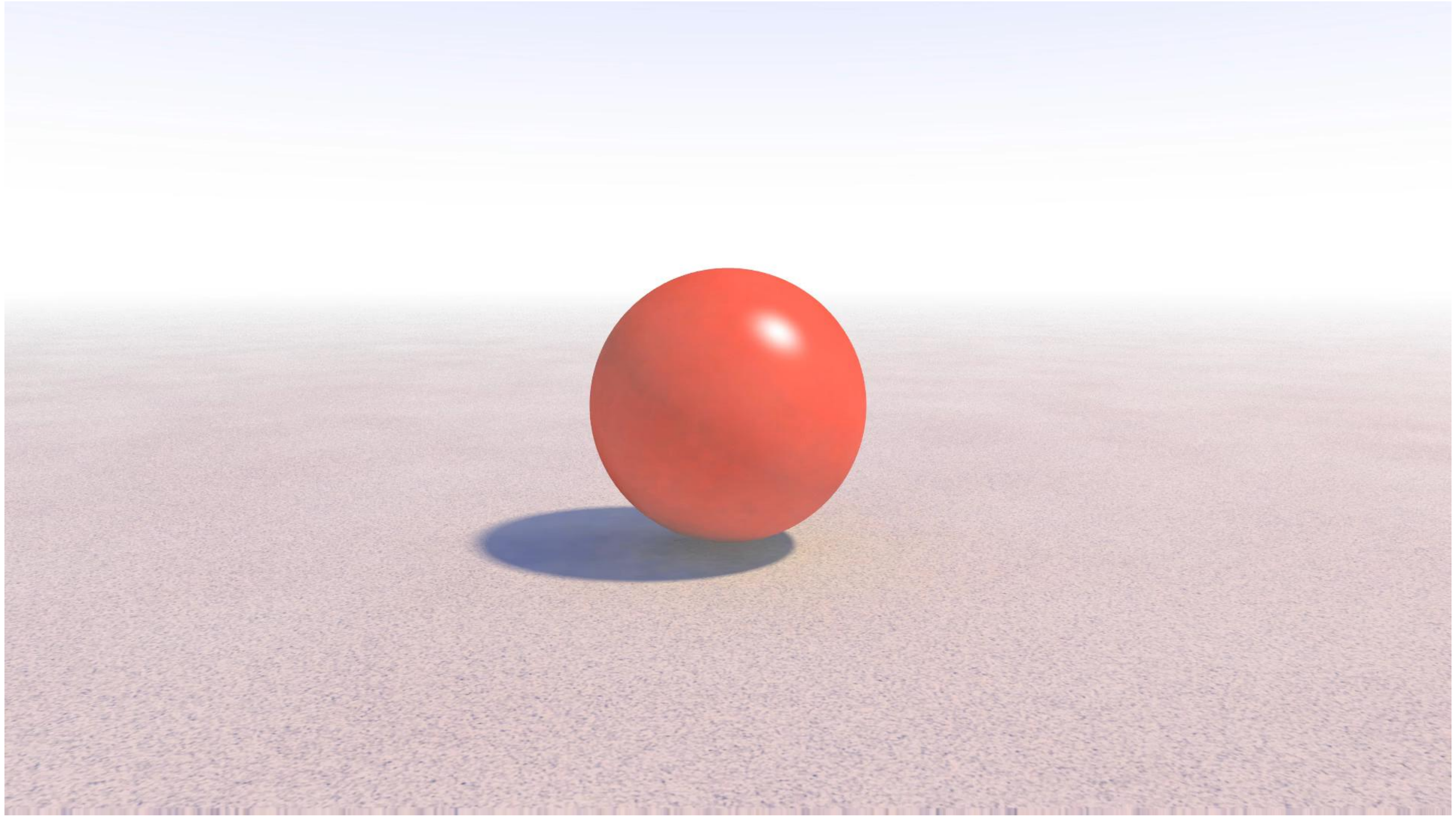
$$F_n = K\delta^n$$

$$K = \frac{4}{3(\sigma_i + \sigma_j)}\left[\frac{R_i R_j}{R_i + R_j}\right]^{0.5}$$

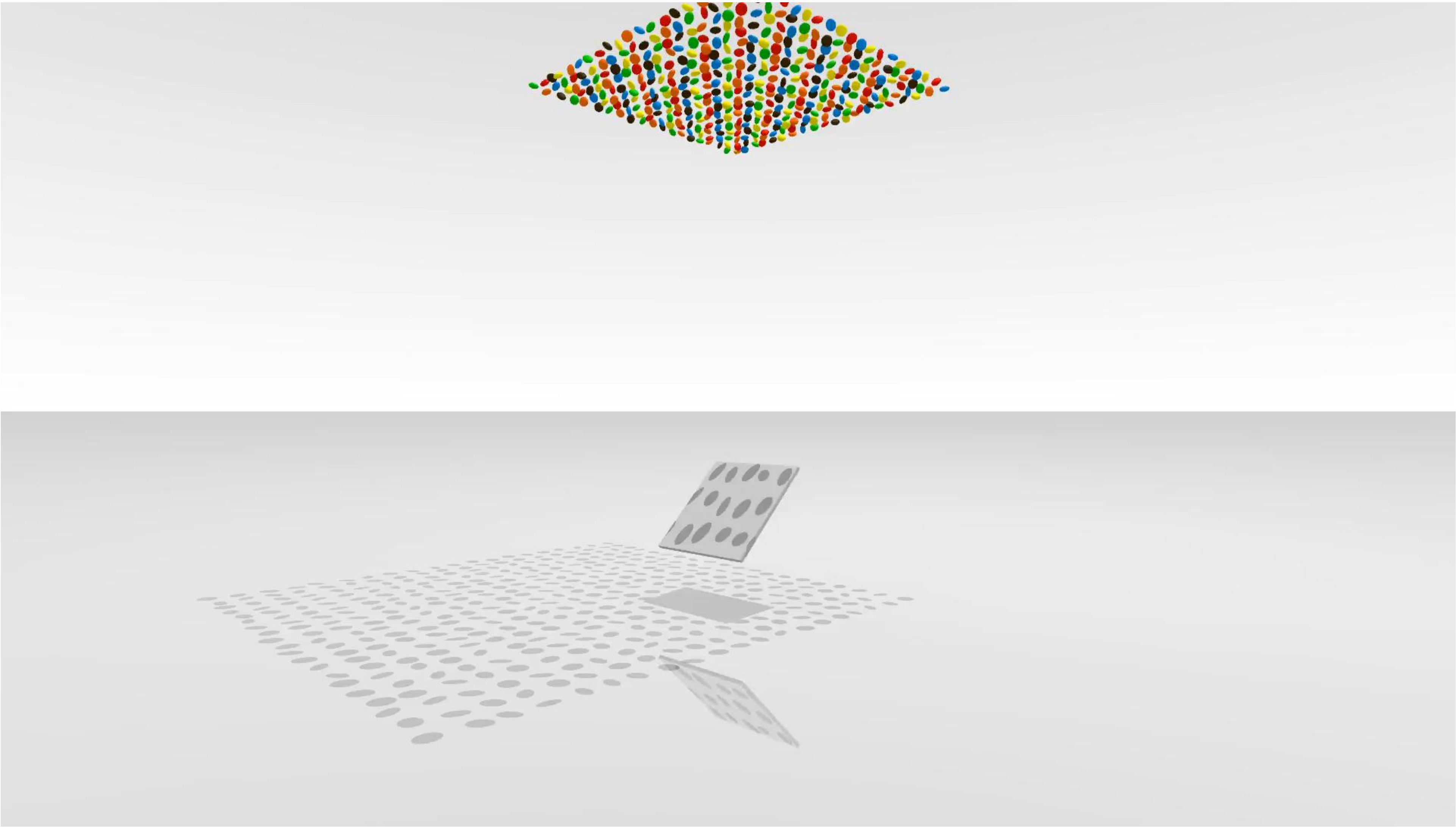


  - Damping and friction can also be introduced
  - Relies on a spherical decomposition of the geometry

Ball – Deformable Net Interaction



Chrono::Rigid - Mixing 50,000 M&Ms on the GPU



Many-Body Dynamics with Friction and Contact

Generalized Positions

Generalized Mass Matrix

Kinematic Differential Equations → $$\dot{\mathbf{q}} = \underbrace{\mathbf{L}(\mathbf{q})}_{\text{Velocity Transformation Matrix}}\underbrace{\mathbf{v}}_{\text{Generalized Velocities}}$$

Force Balance Equations → $$\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} = \underbrace{\mathbf{f}(t,\mathbf{q},\mathbf{v})}_{\text{Applied Force}} - \overbrace{\mathbf{g}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q},t)\boldsymbol{\lambda}}^{\text{Reaction Force}} + \overbrace{\sum_{i=1}^{N_c}\left(\gamma_n^i \mathbf{D}_n^{T,i} + \gamma_u^i \mathbf{D}_u^{T,i} + \gamma_w^i \mathbf{D}_w^{T,i}\right)}^{\text{Frictional Contact Force}}$$

Holonomic Kinematic Constraints → $$\mathbf{g}(\mathbf{q},t) = \mathbf{0}$$

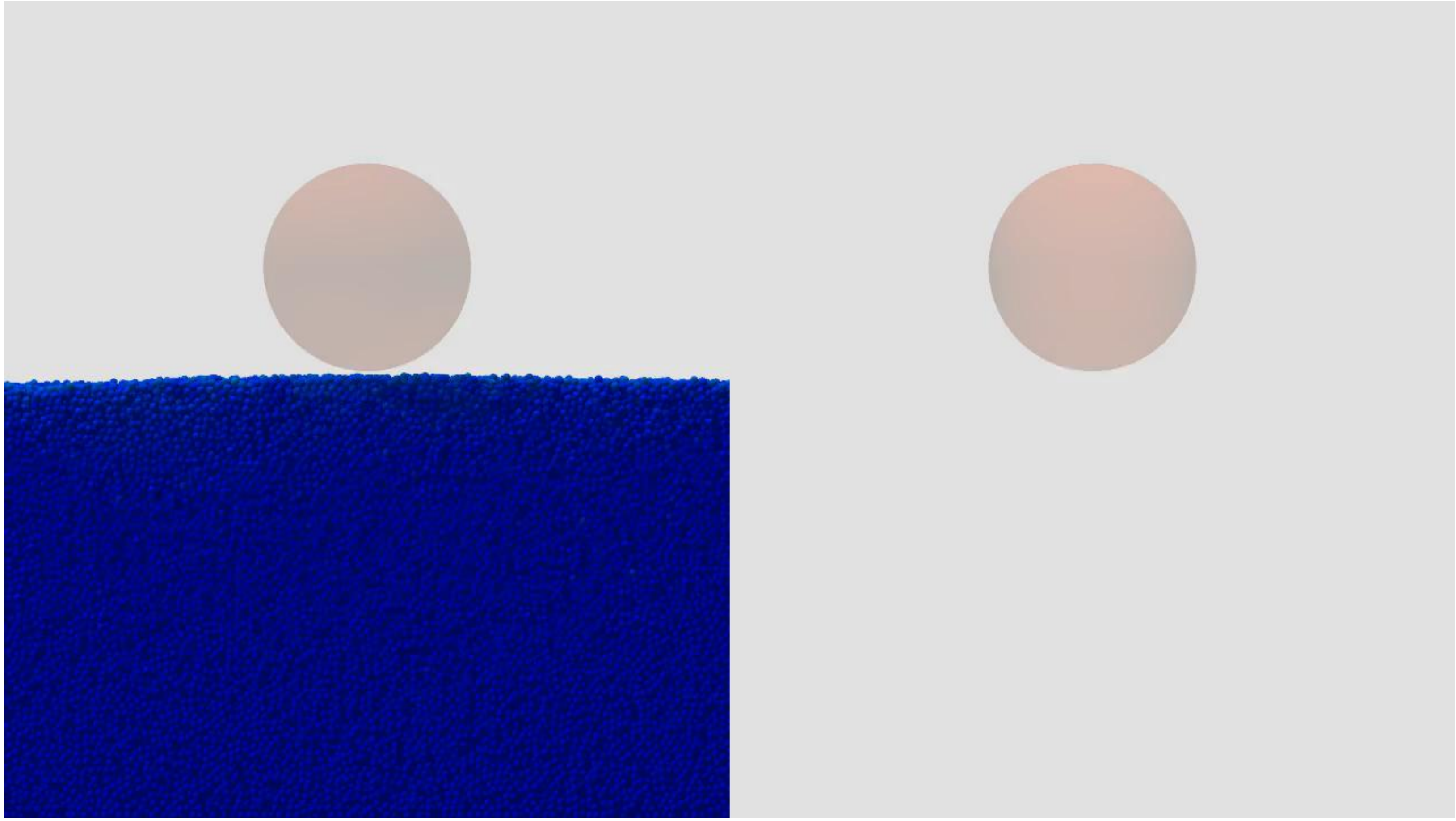
Contact Complementarity Conditions → $$0 \le \Phi^i(\mathbf{q},t) \quad \perp \quad \overbrace{\gamma_n^i}^{\text{Contact Impulse, for Contact ``}i\text{''}} \ge 0 \qquad i = 1,2,\dots,\underbrace{N_c}_{\text{Total Number of Contacts}}$$

Coulomb Friction Model → $$(\gamma_u^i,\gamma_w^i) = \underset{\mu^i\gamma_n^i \ge \sqrt{(\gamma_u^i)^2+(\gamma_w^i)^2}}{\arg\min} \underbrace{\left(\gamma_u^i \mathbf{v}^T \mathbf{D}_u^i + \gamma_w^i \mathbf{v}^T \mathbf{D}_w^i\right)}_{\text{Friction Dissipation Energy}}$$

Gap Function, for Contact "*i*"

Friction Impulse Components, for Contact "*i*"

h=10 cm, ρ_b=2.2 g/cm^3



Chrono::Flow Particle in Suspension Flow

Coupled Problem: Fluid-Solid Interaction

Generalized Positions

Velocity Transformation Matrix

Generalized Mass Matrix

Generalized Velocities

Kinematic Differential Equations → $\dot{\mathbf{q}} = \mathbf{L}(\mathbf{q})\mathbf{v}$

Force Balance Equations → $\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} = \underbrace{\mathbf{f}(t,\mathbf{q},\mathbf{v})}_{\text{Applied Force}} - \overbrace{\mathbf{g}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q},t)\boldsymbol{\lambda}}^{\text{Reaction Force}} + \overbrace{\sum_{i=1}^{N_c}(\gamma_n^i \mathbf{D}_n^{T,i} + \gamma_u^i \mathbf{D}_u^{T,i} + \gamma_w^i \mathbf{D}_w^{T,i})}^{\text{Frictional Contact Force}}$

Holonomic Kinematic Constraints → $\mathbf{g}(\mathbf{q},t) = \mathbf{0}$

Conservation of mass: $\frac{d\rho}{dt} = -\rho \frac{\partial v^\beta}{\partial x^\beta}$

Conservation of momentum: $\frac{dv^\alpha}{dt} = \frac{1}{\rho}\frac{\partial \sigma^{\alpha\beta}}{\partial x^\beta} + \frac{f^\alpha}{\rho}$

Conservation of energy: $\frac{du}{dt} = \frac{\sigma^{\alpha\beta}}{\rho}\frac{\partial v^\alpha}{\partial x^\beta}$

Algorithmic (applied math) support

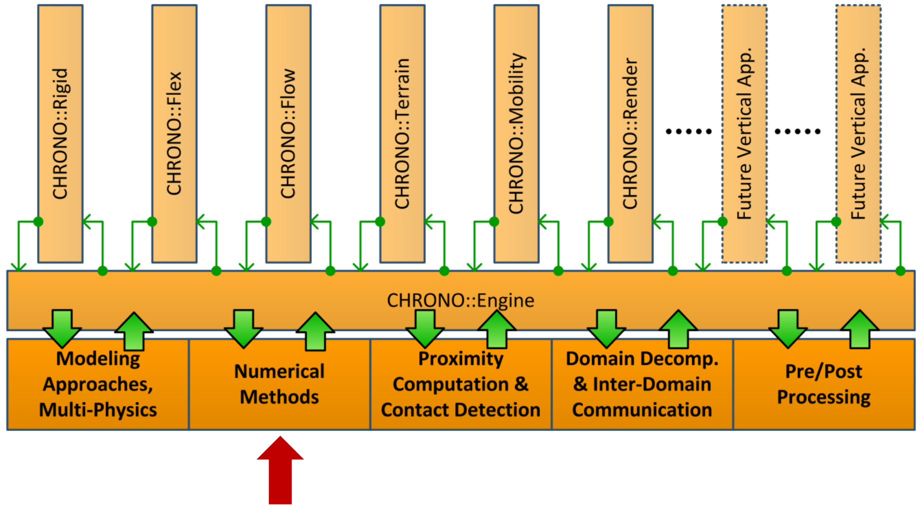
Advanced modeling techniques

Algorithmic (applied math) support

Proximity computation

Domain decomposition & Inter-domain data exchange

Post-processing (visualization)



Deformable Bodies: Implicit Integration using Newmark…

Newmark Integration Formula

- New positions and velocities obtained at t_{n+1} based on new accelerations & Lagrange multipliers

$$\mathbf{q}_{n+1} = \mathbf{q}_n + h\dot{\mathbf{q}}_n + \frac{h^2}{2}\left[(1-2\beta)\ddot{\mathbf{q}}_n + 2\beta\ddot{\mathbf{q}}_{n+1}\right]$$

$$\dot{\mathbf{q}}_{n+1} = \dot{\mathbf{q}}_n + h\left[(1-\gamma)\ddot{\mathbf{q}}_n + \gamma\ddot{\mathbf{q}}_{n+1}\right]$$

Index 3 DAE Problem

- Discretized Equations of Motion at t_{n+1}:

$$(\mathbf{M}\ddot{\mathbf{q}})_{n+1} + (\mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda})_{n+1} + (\mathbf{Q}_{\text{int}} - \mathbf{Q}_{\text{ext}})_{n+1} = 0$$

- Kinematic constraints evaluated at new time step t_{n+1}:

$$\mathbf{\Phi}(\mathbf{q}_{n+1}, t_{n+1}) = 0$$

Deformable Bodies: Implicit Integration using Newmark…

- Solving an index 3 set of Differential Algebraic Equations (DAEs) w/ implicit integration
  - Relies on Newton-Krylov approach to solve nonlinear problem at each time step
  - Updates in the accelerations at iteration (k) computed as

$$\begin{bmatrix} \hat{\mathbf{M}} & \mathbf{\Phi}_{\mathbf{q}}^T \\ \mathbf{\Phi}_{\mathbf{q}} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \Delta\ddot{\mathbf{q}} \\ \Delta\boldsymbol{\lambda} \end{bmatrix}^{(k)} = \begin{bmatrix} \text{-}\mathbf{e}_1 \\ \text{-}\mathbf{e}_2 \end{bmatrix}^{(k)}$$

    - Residuals capture error in satisfying the equations of motion and the kinematic constraint equations:

$$\mathbf{e}_1 = \left(\mathbf{M}\ddot{\mathbf{q}}\right)_{n+1} + (\mathbf{\Phi}_{\mathbf{q}}^T\boldsymbol{\lambda})_{n+1} + (\mathbf{Q}_{\text{int}})_{n+1} - (\mathbf{Q}_{\text{ext}})_{n+1}$$

$$\mathbf{e}_2 = \frac{1}{\beta h^2}\mathbf{\Phi}(\mathbf{q}_{n+1}, t_{n+1})$$

Jacobian Matrix Computation, Flex Body Dynamics

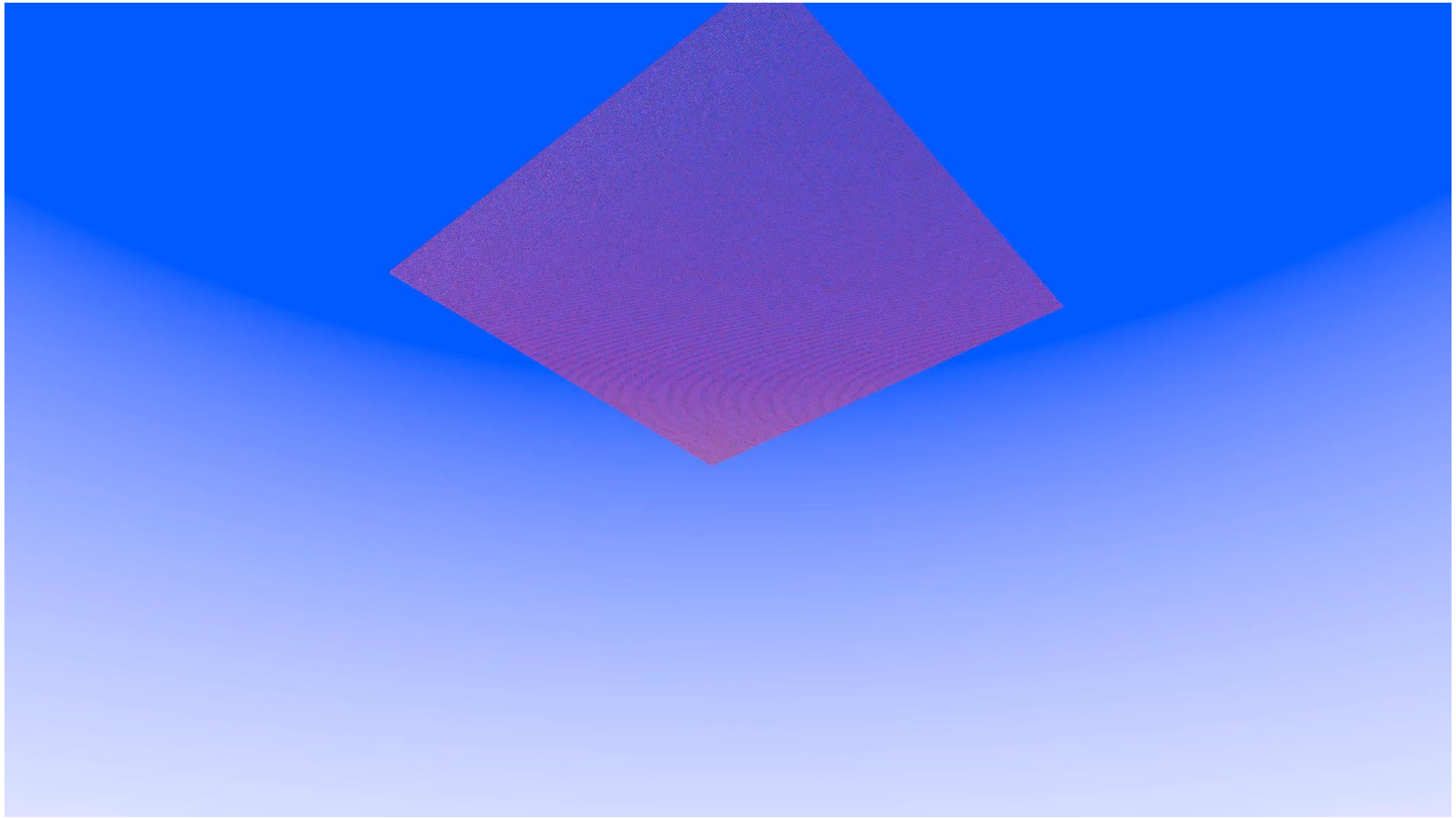
- Sensitivity computation costly

$$\hat{\mathbf{M}} = \frac{\partial \mathbf{e}_1}{\partial \ddot{\mathbf{q}}} = \mathbf{M} - h\gamma \left[\frac{\partial \mathbf{Q}_{ext}}{\partial \dot{\mathbf{q}}} \right] + \beta h^2 \left[(\mathbf{\Phi}_{\mathbf{q}}^T \boldsymbol{\lambda})_{\mathbf{q}} + \frac{\partial \mathbf{Q}_{\text{int}}}{\partial \mathbf{q}} - \frac{\partial \mathbf{Q}_{ext}}{\partial \mathbf{q}} \right]$$

- Computational bottleneck is evaluation of sensitivity of internal forces:

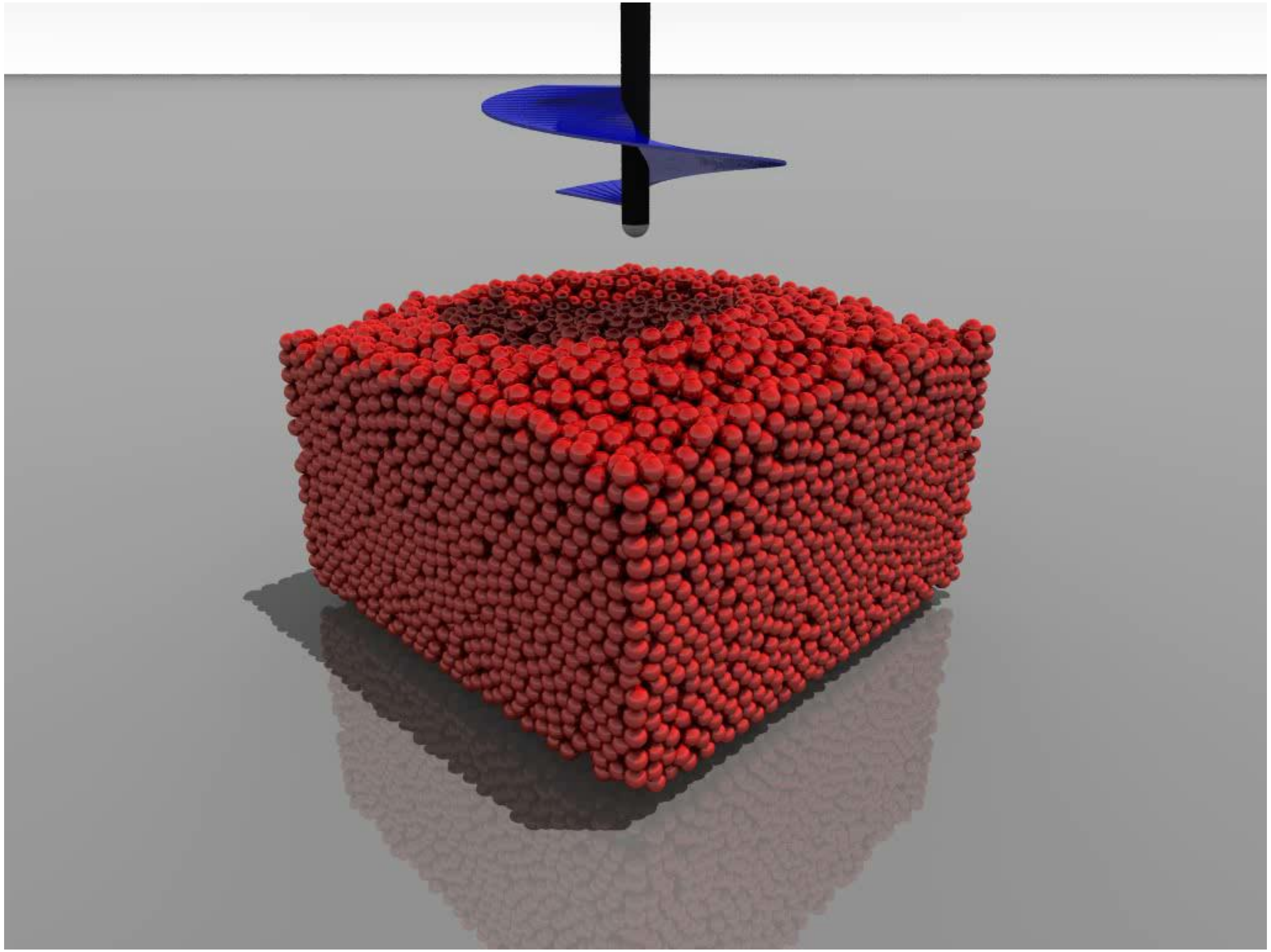
$$\frac{\partial \mathbf{Q}_{\text{int}}}{\partial \mathbf{q}} = \int_0^l EA(\varepsilon_{11}) \frac{\partial}{\partial \mathbf{e}} \left(\frac{\partial \varepsilon_{11}}{\partial \mathbf{e}} \right)^T dx + \int_0^l EA \left(\frac{\partial \varepsilon_{11}}{\partial \mathbf{e}} \right)^T \left(\frac{\partial \varepsilon_{11}}{\partial \mathbf{e}} \right) dx + \int_0^l EI(\kappa) \frac{\partial}{\partial \mathbf{e}} \left(\frac{\partial \kappa}{\partial \mathbf{e}} \right)^T dx + \int_0^l EI \left(\frac{\partial \kappa}{\partial \mathbf{e}} \right)^T \left(\frac{\partial \kappa}{\partial \mathbf{e}} \right) dx$$

0.25 km Net Simulation: 101,025 Beams & 640,146 Constraints

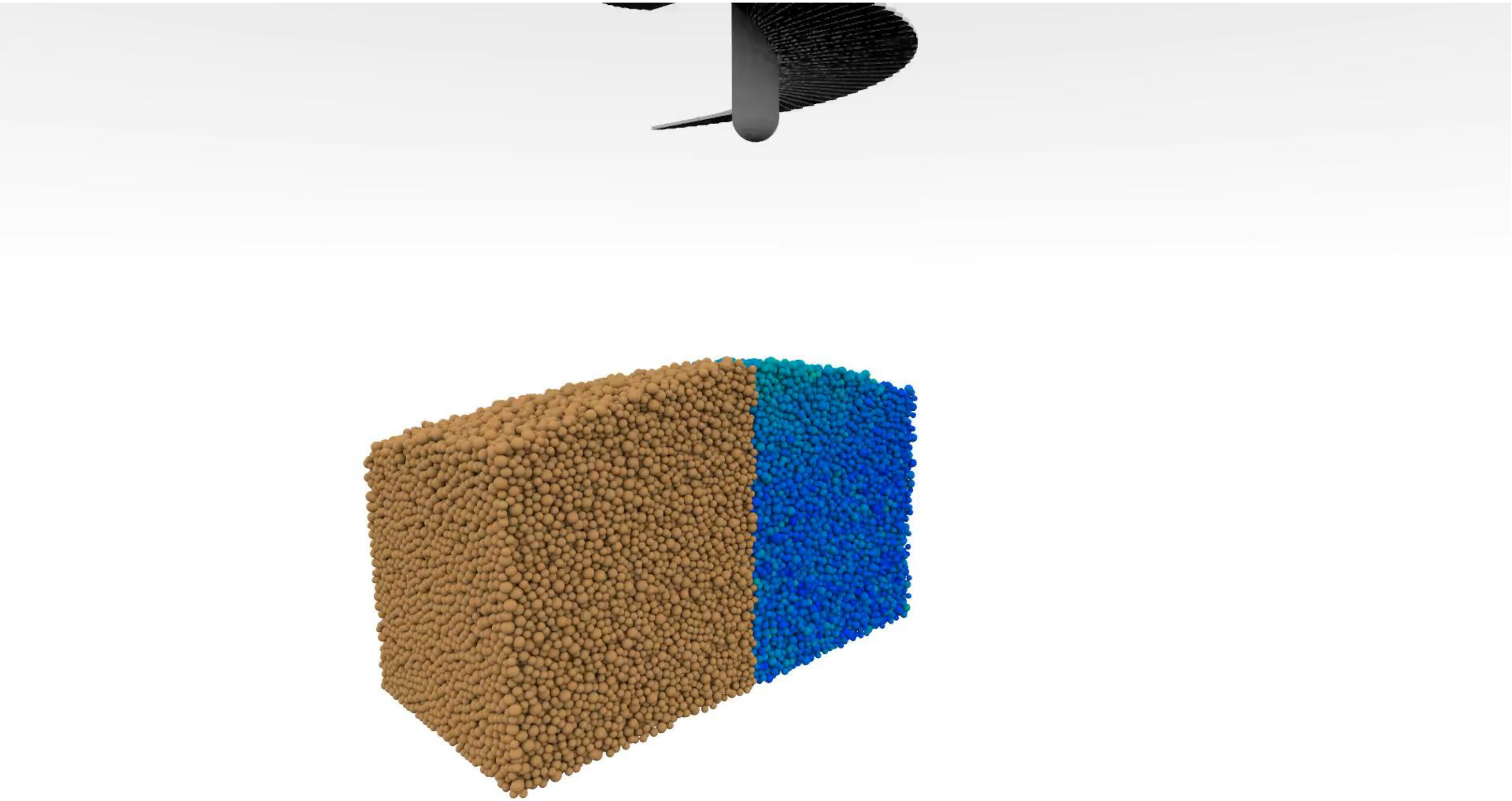


200,000 Bodies & 10 kg Anchor

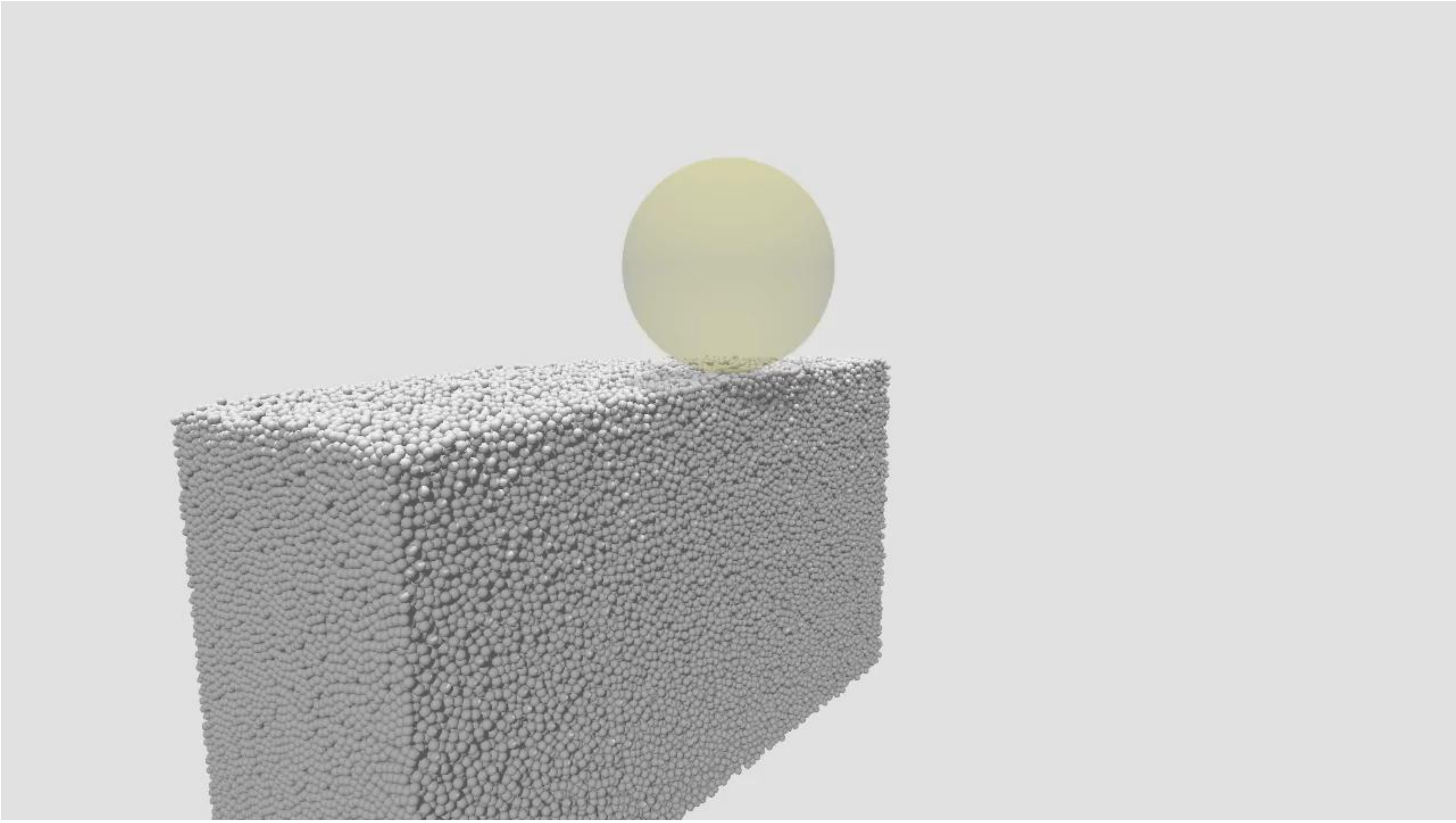
[solved in parallel on the GPU]



Cut-away, Anchoring Simulation

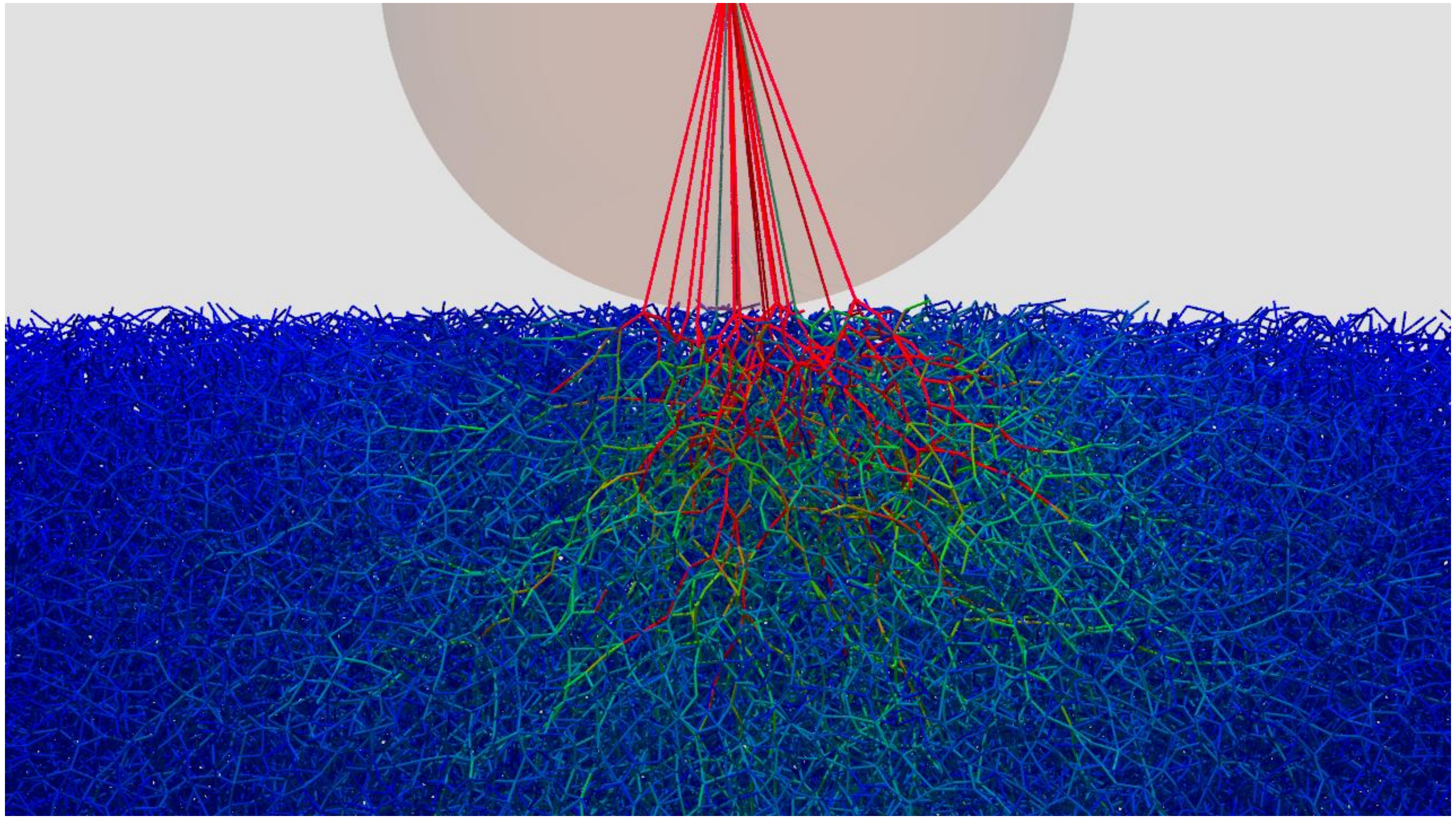


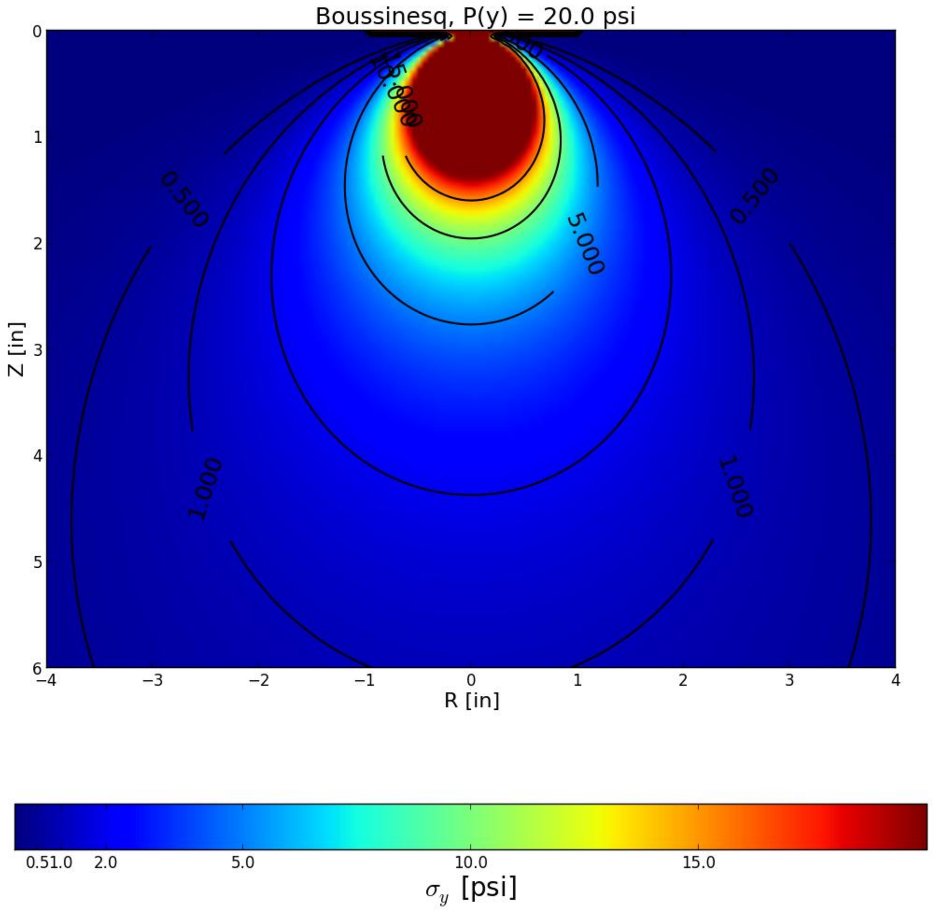
0.5 Million Rigid Bodies



0.5 Million Rigid Bodies

- Granular material has unique properties





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Rigid Body Dynamics w/ DVI: From Continuum to Discrete

- Seeking to solve numerically the equations of motion robustly & effectively
- Discussion focused here on many-body dynamics

Generalized Positions

Velocity Transformation Matrix

Generalized Mass Matrix

Generalized Velocities

Kinematic Differential Equations → $\dot{\mathbf{q}} = \mathbf{T}(\mathbf{q})\mathbf{v}$

Force Balance Equations → $\mathbf{M}(\mathbf{q})\dot{\mathbf{v}} = \underbrace{\mathbf{f}(t,\mathbf{q},\mathbf{v})}_{\text{Applied Force}} - \overbrace{\mathbf{g}_{\mathbf{q}}^{\mathrm{T}}(\mathbf{q},t)\boldsymbol{\lambda}}^{\text{Reaction Force}} + \overbrace{\sum_{i=1}^{N_c}(\gamma_n^i \mathbf{D}_n^{T,i} + \gamma_u^i \mathbf{D}_u^{T,i} + \gamma_w^i \mathbf{D}_w^{T,i})}^{\text{Frictional Contact Force}}$

Holonomic Kinematic Constraints → $\mathbf{g}(\mathbf{q},t) = \mathbf{0}$

Contact Complementarity Conditions → $0 \le \Phi^i(\mathbf{q},t) \perp \overbrace{\gamma_n^i}^{\text{Contact Impulse, for Contact "}i\text{"}} \ge 0 \qquad i = 1,2,\ldots,\underbrace{N_c}_{\text{Total Number of Contacts}}$

Coulomb Friction Model → $(\gamma_u^i, \gamma_w^i) = \underset{\mu^i \gamma_n^i \ge \sqrt{(\gamma_u^i)^2 + (\gamma_w^i)^2}}{\arg\min} \underbrace{(\gamma_u^i \mathbf{v}^T \mathbf{D}_u^i + \gamma_w^i \mathbf{v}^T \mathbf{D}_w^i)}_{\text{Friction Dissipation Energy}}$

Gap Function, for Contact "i"

Friction Impulse Components, for Contact "i"

The Discretized Problem: from t_l to t_{l+1}

positions

time step index

$$\mathbf{q}^{(l+1)} = \mathbf{q}^{(l)} + h\mathbf{L}(\mathbf{q}^{(l)})\mathbf{v}^{(l+1)}$$

Mass Mat.

speeds

Applied Forces

Reaction impulses

$$\mathbf{M}(\mathbf{v}^{(l+1)} - \mathbf{v}^{l}) = h\mathbf{f}(t^{(l)}, \mathbf{q}^{(l)}, \mathbf{v}^{(l)}) + \sum_{i \in \mathcal{A}(q^{(l)}, \delta)} (\gamma_{i,n}\,\mathbf{D}_{i,n} + \gamma_{i,u}\,\mathbf{D}_{i,u} + \gamma_{i,w}\,\mathbf{D}_{i,w})$$

$$i \in \mathcal{A}(q^{(l)}, \delta): \quad 0 \le \frac{1}{h}\Phi_i(\mathbf{q}^{(l)}) + \mathbf{D}_{i,n}^T \mathbf{v}^{(l+1)} \perp \gamma_n^i \ge 0,$$

Complementarity Condition

$$(\gamma_{i,u}, \gamma_{i,w}) = \operatorname*{argmin}_{\mu_i \gamma_{i,n} \ge \sqrt{\gamma_{i,u}^2 + \gamma_{i,w}^2}} \mathbf{v}^T (\gamma_{i,u}\,\mathbf{D}_{i,u} + \gamma_{i,w}\,\mathbf{D}_{i,w}).$$

Stabilization term

Coulomb 3D friction model

(D. Stewart, 1998)

The Cone Complementarity Problem (CCP)

- Define the convex hypercone...

$$\Upsilon = \left(\bigoplus_{i \in \mathcal{A}(\mathbf{q}^l, \epsilon)} \mathcal{FC}^i \right)$$

$\mathcal{FC}^i \in \mathbb{R}^3$ represents friction cone associated with i^{th} contact

- ... and its polar hypercone:

$$\Upsilon^\circ = \left(\bigoplus_{i \in \mathcal{A}(\mathbf{q}^l, \epsilon)} \mathcal{FC}^{i\circ} \right)$$

- The problem can be formulated as find γ that solves the following CCP

$$\gamma \in \Upsilon \perp -(\mathbf{N}\gamma + \mathbf{d}) \in \Upsilon^\circ$$

The Quadratic Programming Angle…

- The CCP captures the first-order optimality condition for a quadratic optimization problem with conic constraints:

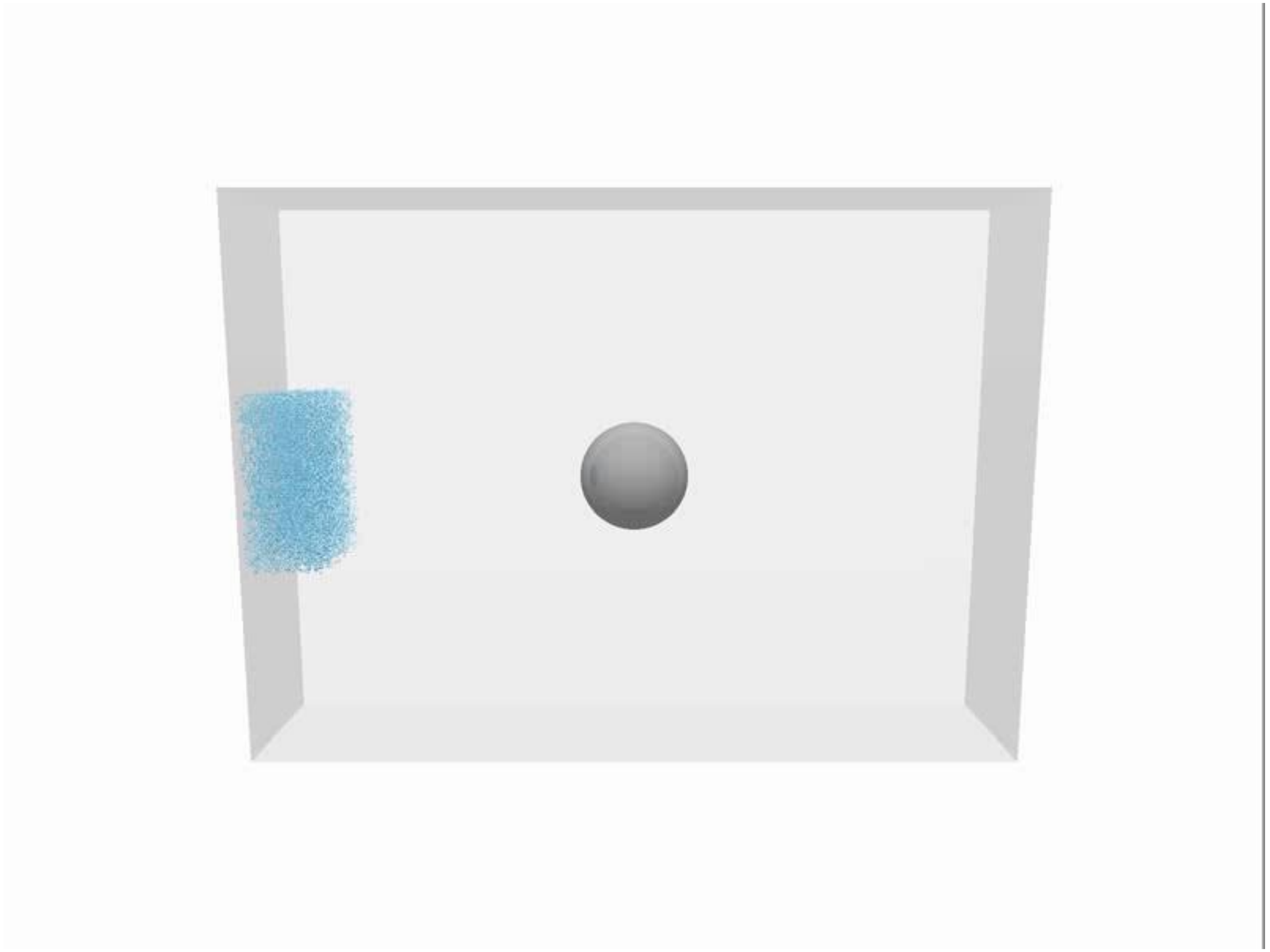
$$\begin{cases} \min \mathbf{q}(\gamma) = \frac{1}{2}\gamma^{\mathbf{T}}\mathbf{N}\gamma + \mathbf{d}^{\mathbf{T}}\gamma \\ \\ \text{subject to} \quad \gamma_i \in \Upsilon_i \;\; \text{for} \;\; i = 1, 2, \ldots, N_c \end{cases}$$

- Notation used:

$$\gamma \equiv [\gamma_1^T, \gamma_2^T, \ldots, \gamma_{N_c}^T]^T \in \mathbb{R}^{3\times N_c} \qquad \text{and} \qquad \Upsilon_i : (\gamma_{u,i}^2 + \gamma_{w,i}^2) - \mu_i^2 \gamma_{n,i}^2 \leq 0$$

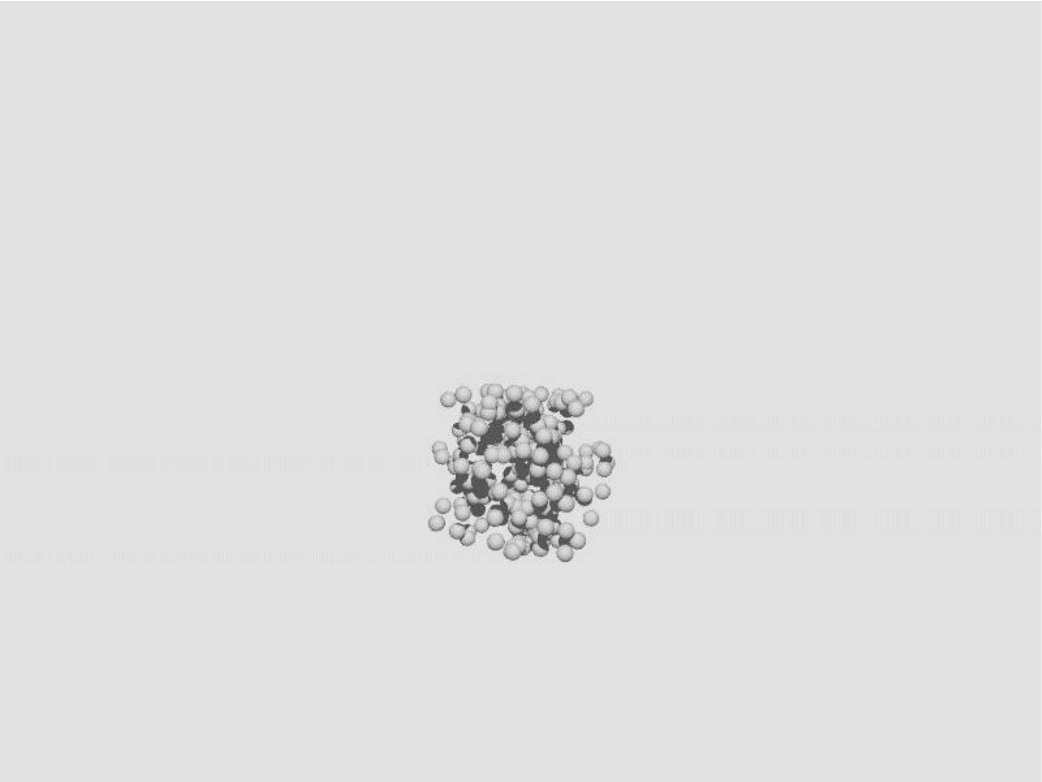
1 Million Rigid Spheres

[parallel on the GPU]

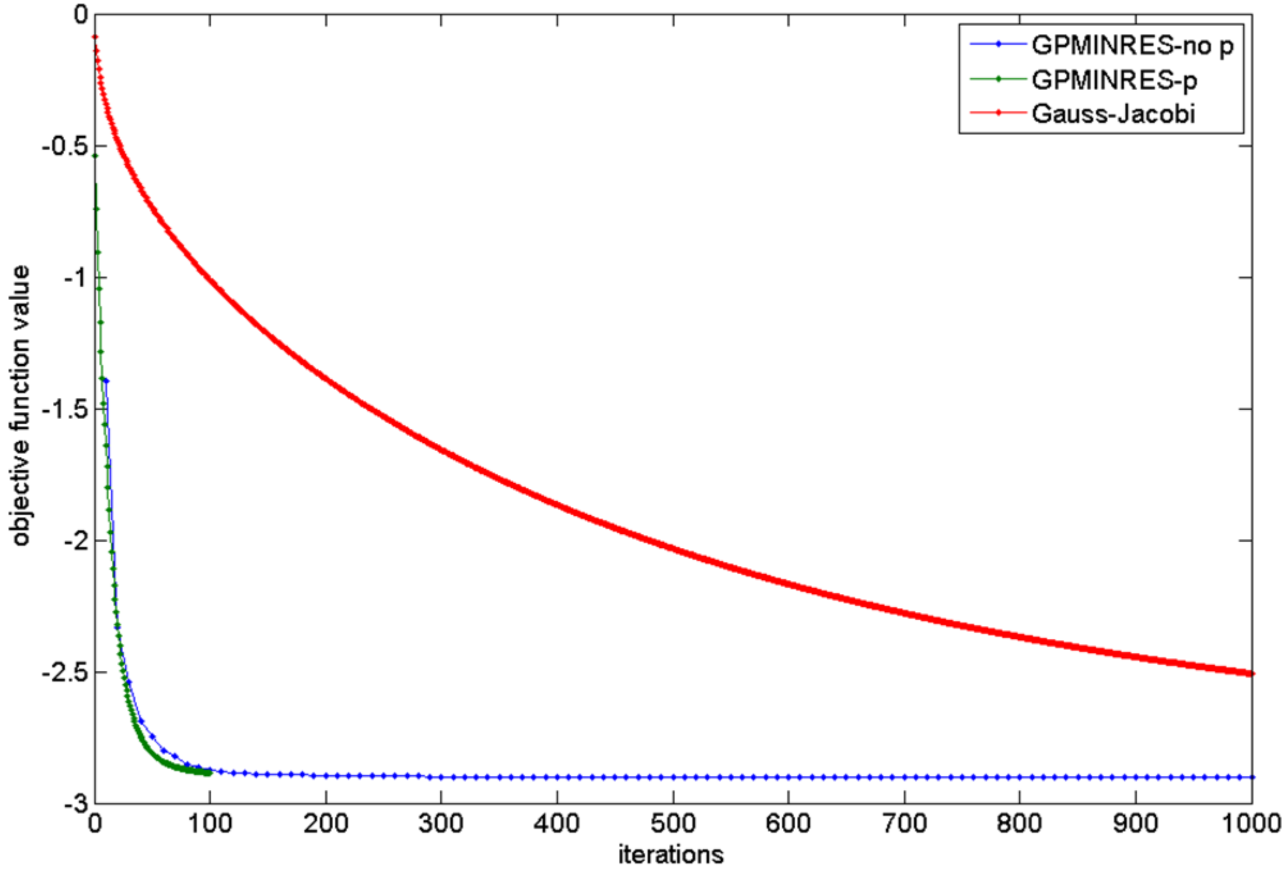


Test Problem: 1000 rigid spheres with 3525 contacts

- Cost function depends on 3,525 variables (or about 10,500 if friction is present)



Objective Function Value [1K bodies, 3525 contacts]



Jacobi

$$
\begin{aligned}
\tilde{\gamma}^{r+1} &= \gamma^r + \omega \mathbf{B}[\mathbf{N}\gamma^r + \mathbf{r}] \\
\hat{\gamma}^{r+1} &= \mathbb{P}(\tilde{\gamma}^{r+1}) \\
\gamma^{r+1} &= \lambda \hat{\gamma}^{r+1} + (1-\lambda)\gamma^r \, .
\end{aligned}
$$

$$
\mathbf{B} = \begin{bmatrix} \eta_1 & & \mathbf{0} \\ & \ddots & \\ \mathbf{0} & & \eta_{n_c} \end{bmatrix} ,
$$

$$
\eta_i = \frac{1}{\mathrm{Trace}(\mathbf{D}_i^T \mathbf{M}^{-1} \mathbf{D}_i)} .
$$

GP-MINRES

ALGORITHM GPMINRES($\mathbf{N}$, $\mathbf{r}$, τ, η_1, η_2, N_{max}, M_{max})

$\gamma^{(0)} := \mathbf{0}_{nc}$
for $k := 0$ **to** N_{max}
  $\mathbf{y}^{(0)} = \gamma^{(k)}$
  while agressively changing active set and reducing cost function
    $\mathbf{y}^{(j+1)} = P[\mathbf{y}^{(j)} - \alpha_j \nabla q(\mathbf{y}^{(j)})]$
    $j = j + 1$
  endwhile
  $\gamma^{(k)} := \mathbf{y}^{(j)}$
  Determine active set $\mathcal{A}(\gamma^{(k)})$ and $\mathbf{Z}_k$ and $\mathbf{r}_k$
  $\mathbf{w}_0 = \mathbf{0}_{m_k}$
  for $j := 0$ **to** M_{max}
    MINRES step: $\mathbf{w}^{(j)} \to \mathbf{w}^{(j+1)}$
    $j = j + 1$
    if slughish convergence
      break
  enfor
  Set $\bar{\mathbf{w}}_k := \mathbf{w}^{(j)}$
  Get $\gamma^{(k+1)} \to$ backtracking line-search with direction $\mathbf{d}_k = \mathbf{Z}_k \bar{\mathbf{w}}_k$
  if $||\nabla_\Omega q(\gamma^{(k+1)})||_\infty < \tau$
    break
enfor
return Value at time step t_{l+1}, $\gamma^{l+1} := \gamma^{(k+1)}$.

P-SPG-FB

ALGORITHM P-SPG-FB($\mathbf{N}$, $\mathbf{r}$, $\mathbf{x}_0$, $\mathcal{K}$, $\mathbf{P} \mapsto \mathbf{x}$)

$\mathbf{x}_0 := \Pi_\mathcal{K}(\mathbf{x_0})$, $\mathbf{x}_{FB} = \mathbf{x}_0$, $\hat{\alpha}_0 \in [\alpha_{min}, \alpha_{max}]$

$\mathbf{g}_0 := \mathbf{N}\mathbf{x}_0 + \mathbf{r}$, $f(\mathbf{x}_0) = \frac{1}{2}\mathbf{x}_0^T\mathbf{N}\mathbf{x}_0 + \mathbf{x}_0^T\mathbf{r}$, $w_0 = 10^{29}$

for $j := 0$ **to** N_{max}

$\quad \mathbf{p}_j = \mathbf{P}^{-1}\mathbf{g_j}$

$\quad \mathbf{d}_j = \Pi_\mathcal{K}(\mathbf{x}_j - \hat{\alpha}_j\mathbf{p}_j) - \mathbf{x}_j$

$\quad$ **if** $\langle \mathbf{d}_j, \mathbf{g}_j \rangle \geq 0$

$\qquad \mathbf{d}_j = \Pi_\mathcal{K}(\mathbf{x}_j - \hat{\alpha}_j\mathbf{g}_j) - \mathbf{x}_j$

$\quad \lambda := 1$

$\quad$ **while** line search

$\qquad \mathbf{x}_{j+1} := \mathbf{x}_j + \lambda\mathbf{d}_j$

$\qquad \mathbf{g}_{j+1} := \mathbf{N}\mathbf{x}_{j+1} + \mathbf{r}$

$\qquad f(\mathbf{x}_{j+1}) = \frac{1}{2}\mathbf{x}_{j+1}^T\mathbf{N}\mathbf{x}_{j+1} + \mathbf{x}_{j+1}^T\mathbf{r}$

$\qquad$ **if** $f(\mathbf{x}_{j+1}) > \max_{i=0,..,\min(j,N_{GLL})} f(\mathbf{x}_{j-i}) + \gamma\lambda\langle \mathbf{d}_j, \mathbf{g}_j \rangle$

$\qquad\quad$ define $\lambda_{\text{new}} \in [\sigma_{\min}\lambda, \sigma_{\max}\lambda]$ and repeat line search

$\qquad$ **else**

$\qquad\quad$ terminate line search

$\quad \mathbf{s}_j = \mathbf{x}_{j+1} - \mathbf{x}_j$

$\quad \mathbf{y}_j = \mathbf{g}_{j+1} - \mathbf{g}_j$

$\quad$ **if** j is odd

$\qquad \hat{\alpha}_{j+1} = \frac{\langle \mathbf{s}_j, \mathbf{P}\mathbf{s_j} \rangle}{\langle \mathbf{s}_j, \mathbf{y}_j \rangle}$

$\quad$ **else**

$\qquad \hat{\alpha}_{j+1} = \frac{\langle \mathbf{s}_j, \mathbf{y}_j \rangle}{\langle \mathbf{y}_j, \mathbf{P}^{-1}\mathbf{y_j} \rangle}$

$\quad \hat{\alpha}_{j+1} = \min(\alpha_{\max}, \max(\alpha_{\min}, \hat{\alpha}_{j+1}))$

$\quad w_{j+1} = ||[\mathbf{x}_{j+1} - \Pi_\mathcal{K}(\mathbf{x}_{j+1} - \tau_g\mathbf{g}_{j+1})]/\tau_g||_2 = ||\epsilon||_2$

$\quad$ **if** $w_{j+1} \leq \min_{k=0,..,j} w_k$

$\qquad \mathbf{x}_{FB} = \mathbf{x}_{j+1}$

return $\mathbf{x}_{FB}$

Kucera Algorithm

ALGORITHM KUCERA($\mathbf{N}$, $\mathbf{r}$, $\mathbf{x}^0$, $\mathcal{K}$, $\Gamma > 0$, $\tilde{\alpha} \in (0, \|\mathbf{N}\|^{-1}]$, $\epsilon > 0$)

$$
\begin{array}{l}
k = 0 \\
\mathbf{g} = \mathbf{N}\mathbf{x}^0 + \mathbf{r} \\
\mathbf{p} = \phi(\mathbf{x}^0) \\
\textbf{while } \|\tilde{\mathbf{g}}(\mathbf{x}^k)\| > \epsilon \\
\quad \textbf{if } \tilde{\beta}(\mathbf{x}^k)^T \mathbf{g}(\mathbf{x}^k) \leq \Gamma^2 \tilde{\phi}(\mathbf{x}^k)^T \mathbf{g}(\mathbf{x}^k) \\
\quad\quad \alpha_{cg} = \mathbf{g}^T \mathbf{p} / \mathbf{p}^T \mathbf{N} \mathbf{p} \\
\quad\quad \alpha_f = min(\alpha_{f,i}) \text{ where } \alpha_{f,i} = \begin{cases} \mathbf{x}_i / \mathbf{p}_i, & \text{if } \mathbf{p}_i > 0 \\ \infty, & \text{if } \mathbf{p}_i \leq 0 \end{cases} \\
\quad\quad \textbf{if } \alpha_{cg} < \alpha_f \\
\quad\quad\quad \mathbf{x}^{k+1} = \mathbf{x}^k - \alpha_{cg} \mathbf{p} \\
\quad\quad\quad \mathbf{g} = \mathbf{g} - \alpha_{cg} \mathbf{N} \mathbf{p} \\
\quad\quad\quad \gamma = \phi(\mathbf{x}^{k+1})^T \mathbf{N} \mathbf{p} / \mathbf{p}^T \mathbf{A} \mathbf{p} \\
\quad\quad\quad \mathbf{p} = \phi(\mathbf{x}^{k+1}) - \gamma \mathbf{p} \\
\quad\quad \textbf{else} \\
\quad\quad\quad \mathbf{x}^{k+1/2} = \mathbf{x}^k - \alpha_f \mathbf{p} \\
\quad\quad\quad \mathbf{x}^{k+1} = \mathbf{x}^{k+1/2} - \tilde{\alpha} \tilde{\phi}(\mathbf{x}^{k+1/2}) \\
\quad\quad\quad \mathbf{g} = \mathbf{N} \mathbf{x}^{k+1} + \mathbf{r} \\
\quad\quad\quad \mathbf{p} = \phi(\mathbf{x}^{k+1}) \\
\quad \textbf{else} \\
\quad\quad \mathbf{x}^{k+1} = \mathbf{x}^k - \tilde{\alpha} \tilde{\beta}(\mathbf{x}^k) \\
\quad\quad \mathbf{g} = \mathbf{N} \mathbf{x}^{k+1} + \mathbf{r} \\
\quad\quad \mathbf{p} = \phi(\mathbf{x}^{k+1}) \\
\quad k = k + 1 \\
\textbf{return } \mathbf{x}^k
\end{array}
$$

Nesterov's Accelerated Projected Gradient Descent

ALGORITHM NAPG($\boldsymbol{N}, \boldsymbol{r}, t \leq \frac{1}{\lambda_{max}(\boldsymbol{N})}, \tau, N_{max}$)

$\boldsymbol{\gamma}_0 = \boldsymbol{0}_{n_c}$

$\hat{\boldsymbol{\gamma}}_0 = \boldsymbol{1}_{n_c}$

$\boldsymbol{y}_0 = \boldsymbol{\gamma}_0$

$\theta_0 = 1$

for $k := 0$ **to** N_{max}

    $\boldsymbol{g} = \boldsymbol{N}\boldsymbol{y}_k - \boldsymbol{r}$

    $\boldsymbol{\gamma}_{k+1} = \Pi_{\mathcal{K}}\left(\boldsymbol{y}_k - t\boldsymbol{g}\right)$

    $\theta_{k+1} = \frac{-\theta_k^2 + \theta_k\sqrt{\theta_k^2+4}}{2}$

    $\beta_{k+1} = \theta_k \frac{1-\theta_k}{\theta_k^2+\theta_{k+1}}$

    $\boldsymbol{y}_{k+1} = \boldsymbol{\gamma}_{k+1} + \beta_{k+1}\left(\boldsymbol{\gamma}_{k+1} - \boldsymbol{\gamma}_k\right)$

    $\epsilon = \epsilon\left(\boldsymbol{\gamma}_{k+1}\right)$

    if $\epsilon < \tau$

        break

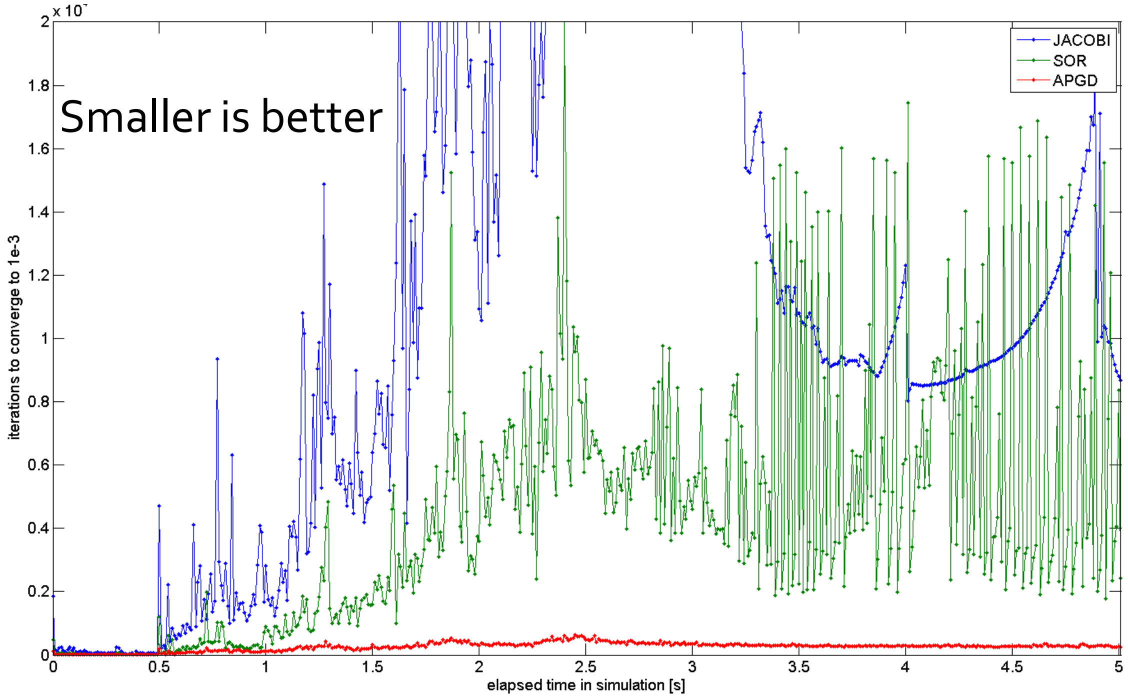
    endif

endfor

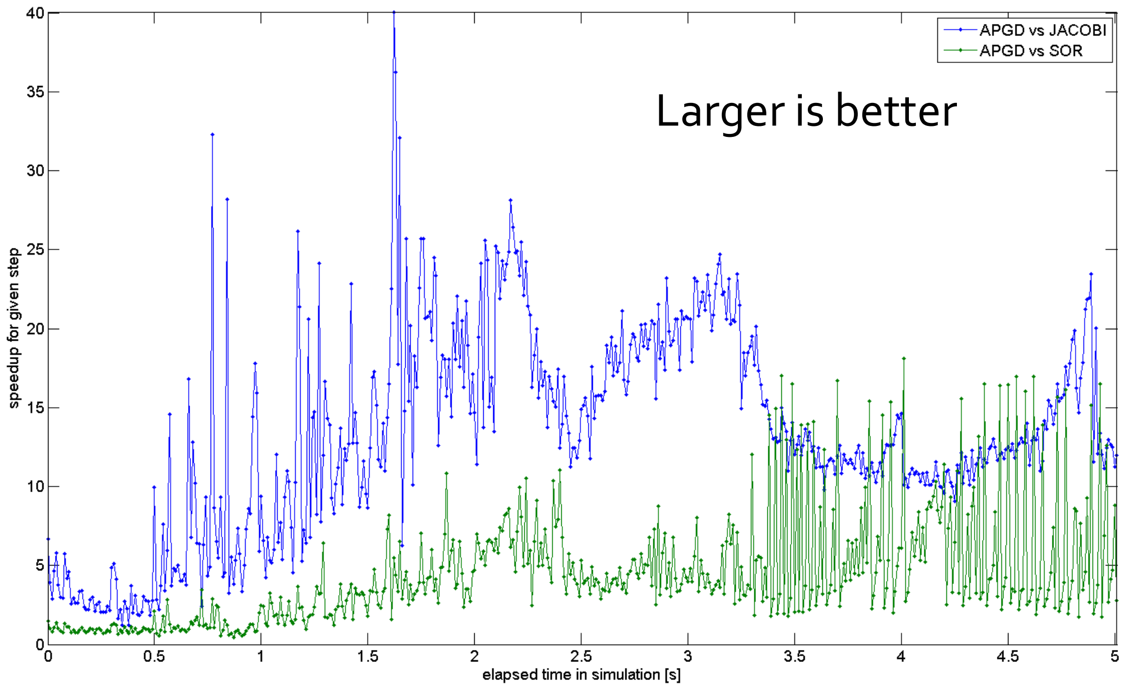
return Value at time step t_{l+1}, $\boldsymbol{\gamma}^{l+1} := \hat{\boldsymbol{\gamma}}$.

Relative Speedup: Benchmark Problem [1000 bodies]

Number of iterations to convergence



Speedup: APGD vs. Jacobi and SOR



Proximity computation

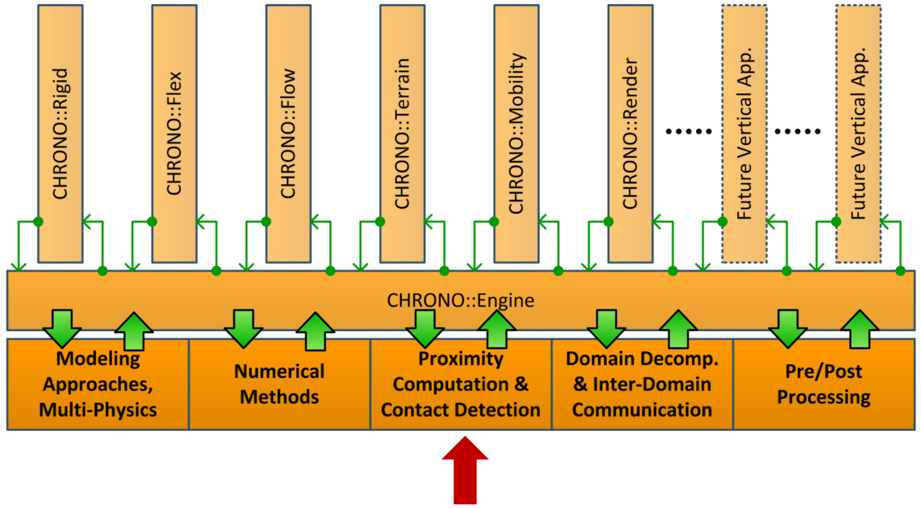
Advanced modeling techniques

Algorithmic (applied math) support

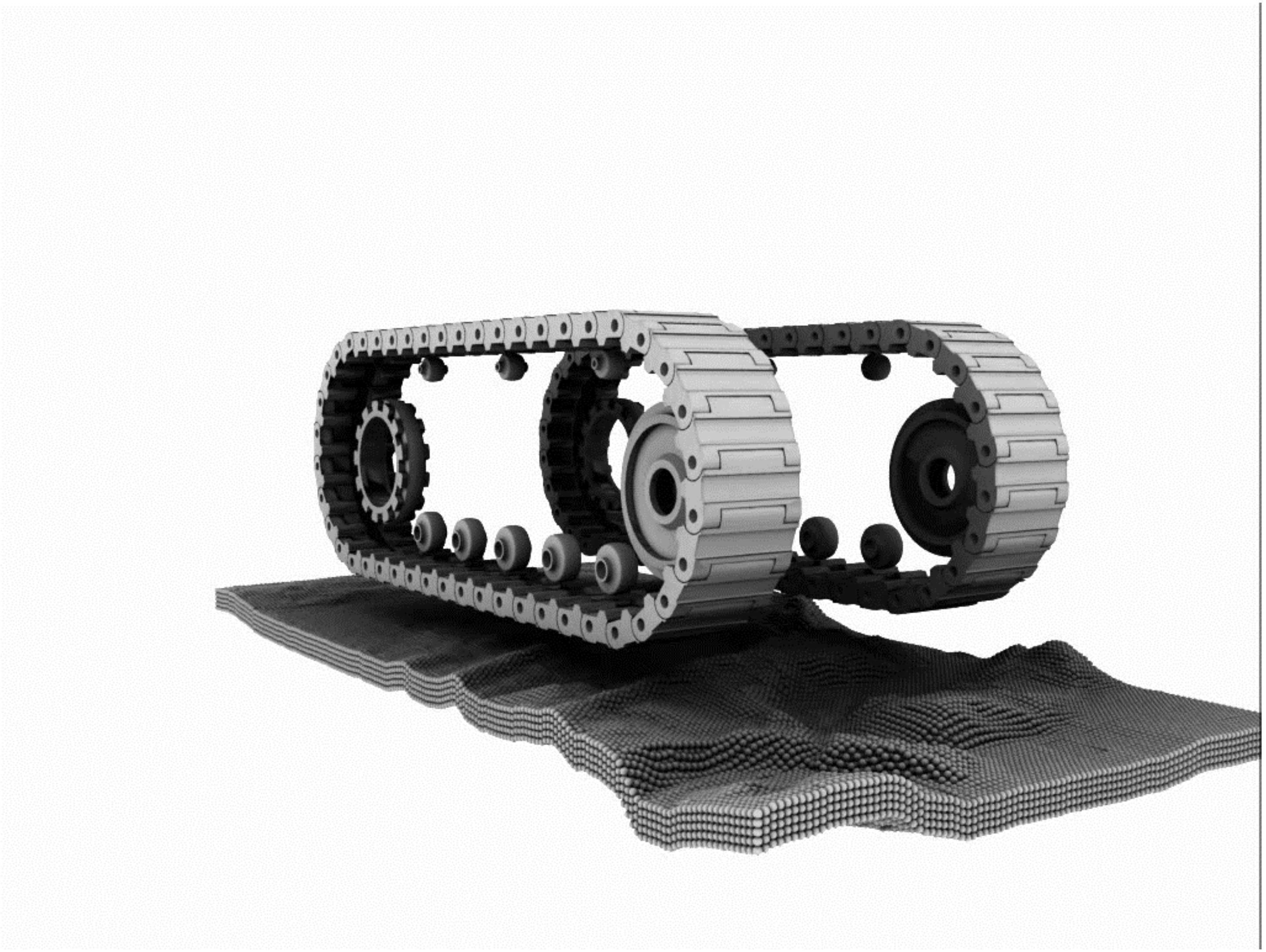
Proximity computation

Domain decomposition & Inter-domain data exchange

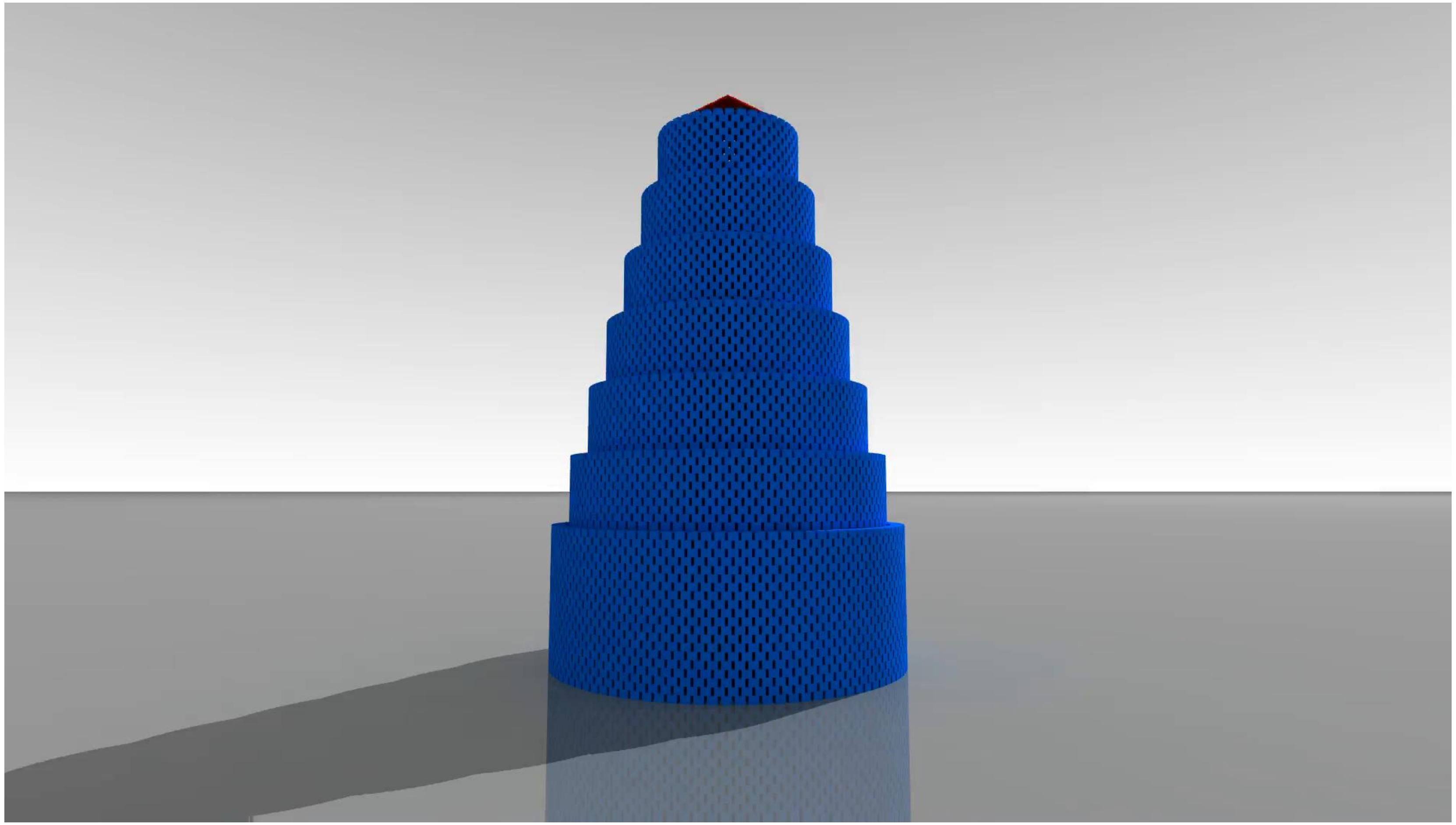
Post-processing (visualization)



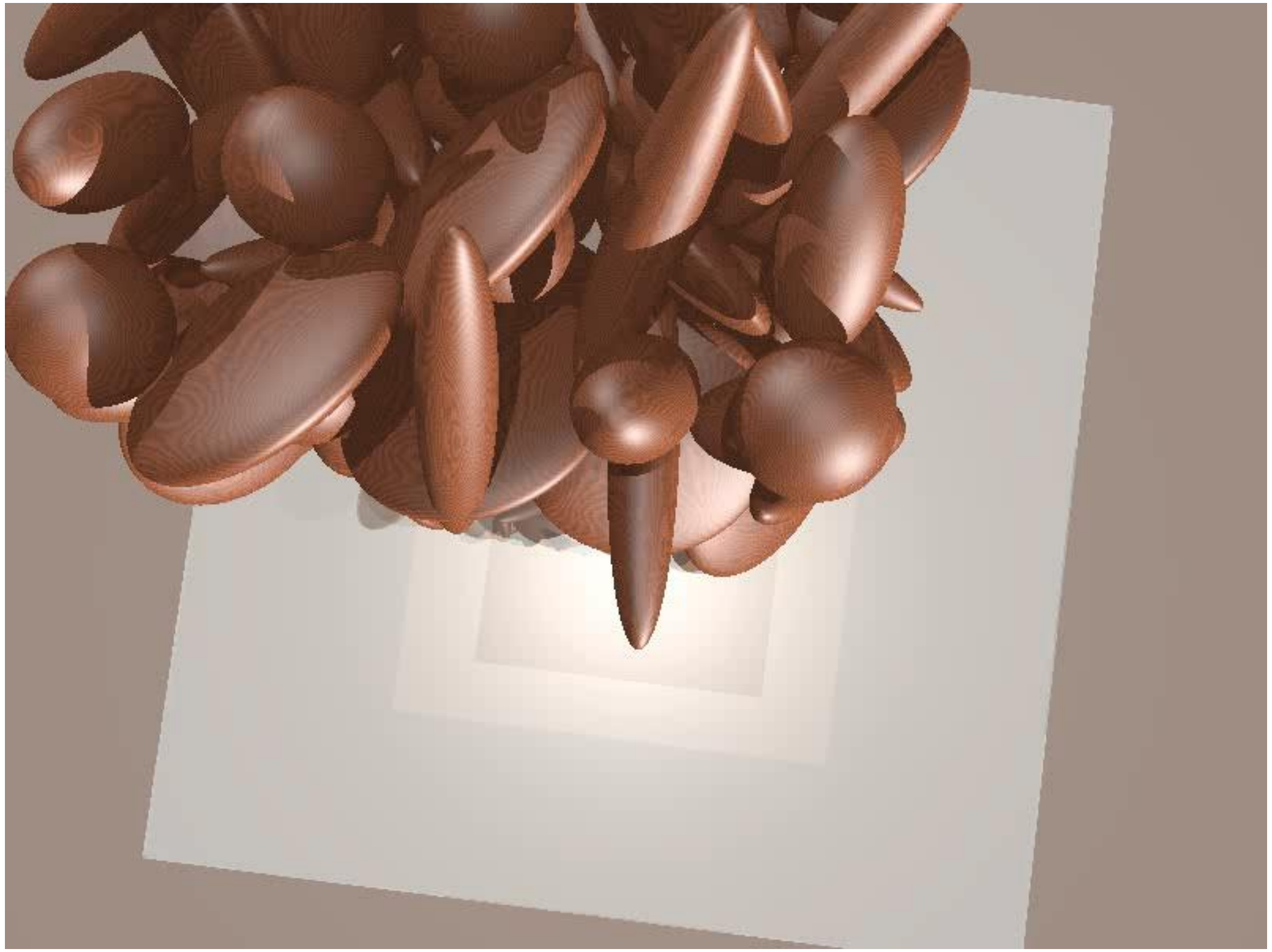
Tracked Vehicle Simulation



600,000 Bodies Moving & Colliding
[run on the GPU]



Ellipsoid-Ellipsoid CD



Various Geometries Handled… [Ellipsoid-Ellipsoid Example]

$$\mathbf{d} = \mathbf{P}_1 - \mathbf{P}_2 = (\frac{1}{2\lambda_1}\mathbf{M}_1 + \frac{1}{2\lambda_2}\mathbf{M}_2)\mathbf{c} + (\mathbf{b}_1 - \mathbf{b}_2)$$

$$\frac{\partial \mathbf{d}}{\partial \alpha_i} = \frac{\partial \mathbf{P}_1}{\partial \alpha_i} - \frac{\partial \mathbf{P}_2}{\partial \alpha_i} \quad , \quad \frac{\partial^2 \mathbf{d}}{\partial \alpha_i \partial \alpha_j} = \frac{\partial^2 \mathbf{P}_1}{\partial \alpha_i \partial \alpha_j} - \frac{\partial^2 \mathbf{P}_2}{\partial \alpha_i \partial \alpha_j}$$

$$\boxed{\frac{\partial \mathbf{P}}{\partial \alpha_i} = (\frac{1}{2\lambda}\mathbf{M} - \frac{1}{8\lambda^3}\mathbf{M}\mathbf{c}\mathbf{c}^T\mathbf{M})\frac{\partial \mathbf{c}}{\partial \alpha_i}}$$

$$\boxed{\begin{aligned}\frac{\partial^2 \mathbf{P}}{\partial \alpha_i \partial \alpha_j} &= (-\frac{1}{8\lambda^3}\mathbf{M} + \frac{3}{32\lambda^5}\mathbf{M}\mathbf{c}\mathbf{c}^T\mathbf{M})\mathbf{c}^T\mathbf{M}\frac{\partial \mathbf{c}}{\partial \alpha_j}\frac{\partial \mathbf{c}}{\partial \alpha_i} \\ &\quad - \frac{1}{8\lambda^3}[(\mathbf{c}^T\mathbf{M}\frac{\partial \mathbf{c}}{\partial \alpha_i})\mathbf{M} + \mathbf{M}\mathbf{c}(\frac{\partial \mathbf{c}}{\partial \alpha_i})^T\mathbf{M}]\frac{\partial \mathbf{c}}{\partial \alpha_j} \\ &\quad + (\frac{1}{2\lambda}\mathbf{M} - \frac{1}{8\lambda^3}\mathbf{M}\mathbf{c}\mathbf{c}^T\mathbf{M})\frac{\partial^2 \mathbf{c}}{\partial \alpha_i \partial \alpha_j}\end{aligned}}$$

$$\varepsilon: \ \frac{x^2}{r_1^2} + \frac{y^2}{r_2^2} + \frac{z^2}{r_3^2} = 1$$

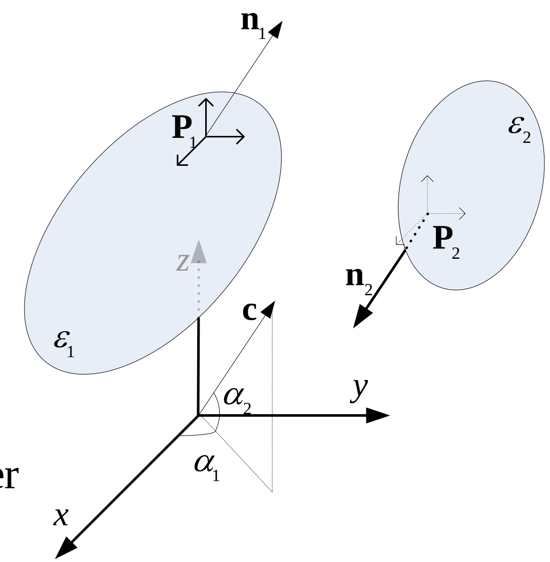
$\mathbf{A}$: Rotation Matrix

$$\mathbf{M} = \mathbf{A}\mathbf{R}^2\mathbf{A}^T$$

$$\mathbf{R} = diag(r_1, r_2, r_3)$$

$\mathbf{b}$: Translation of ellipsoids center

$$\lambda^2 = \frac{1}{4}\mathbf{n}^T\mathbf{M}\mathbf{n}$$



$$\mathbf{d} = \mathbf{P}_1 - \mathbf{P}_2$$

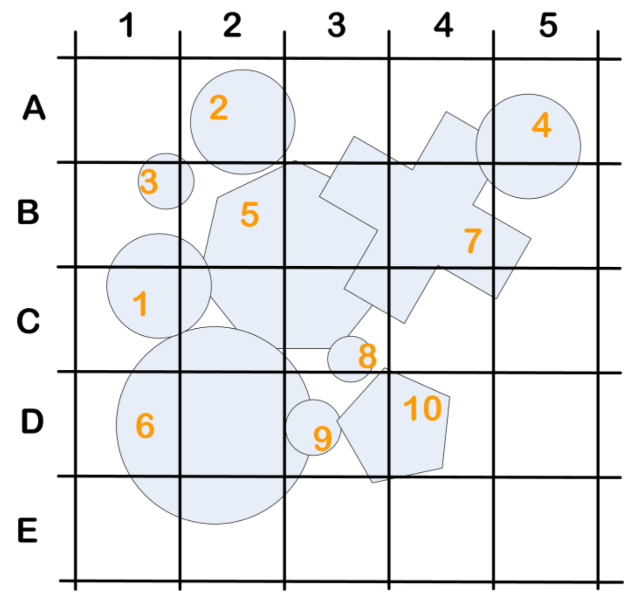
$$\min_{\alpha_1, \alpha_2} \|d(\alpha_1, \alpha_2)\|^2$$

Collision Detection

- Broad phase
  - Draws on an Axis Aligned Bounding Box (AABB) approach
- Narrow phase
  - Draws on Minkowski Portal Refinement

CD: Binning

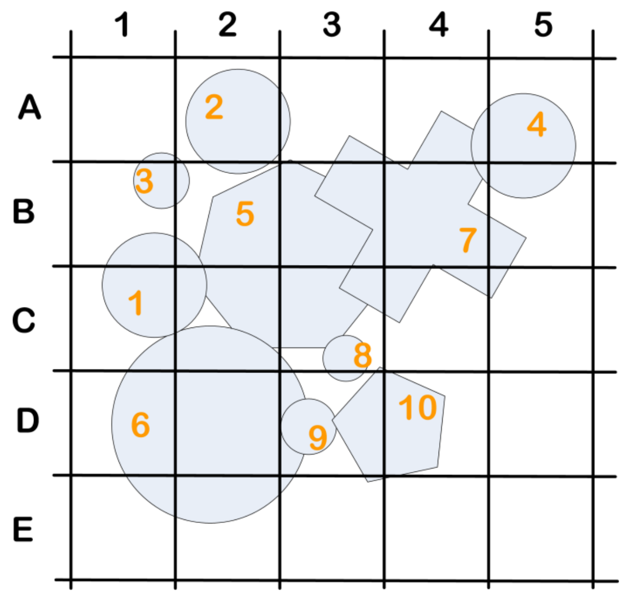
- Example: 2D collision detection, bins are squares

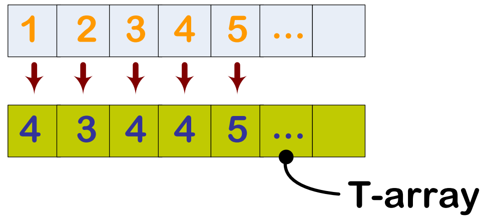


- Body 4 touches bins A4, A5, B4, B5
- Body 7 touches bins A3, A4, A5, B3, B4, B5, C3, C4, C5
- In proposed algorithm, bodies 4 and 7 will be checked for collision by three threads (associated with bin A4, A5, B4)

Stage 1 (Body Parallel)

- Purpose: find the number of bins touched by each body
- Store results in the "**T**", array of N integers
- Key observation: it's easy to bin bodies



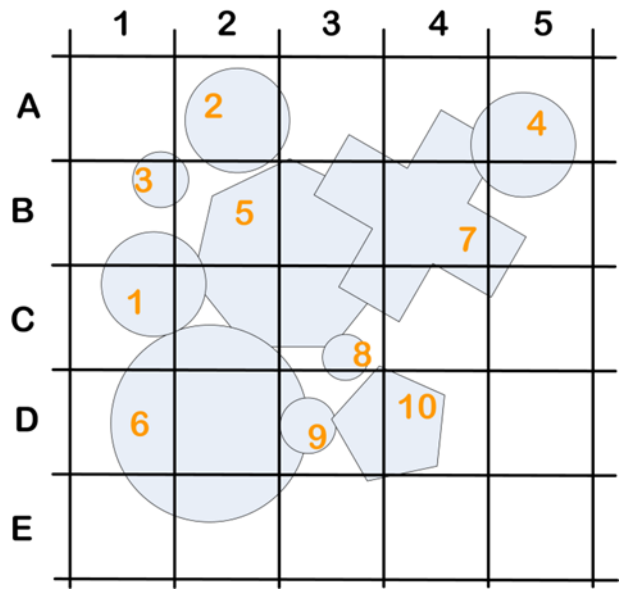


Stage 2: Parallel Inclusive Scan

- Run a parallel inclusive scan on the array **T**
  - The last element is the total number of bin touches, including the last body

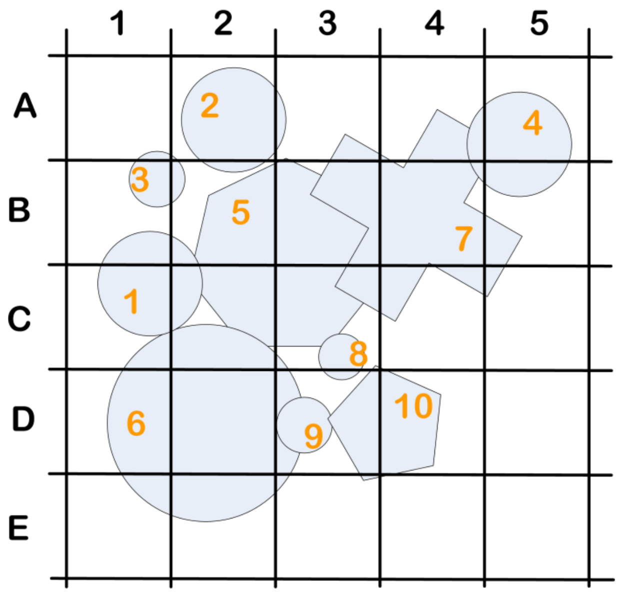
- Complexity of Stage: O(N) – using **thrust** library

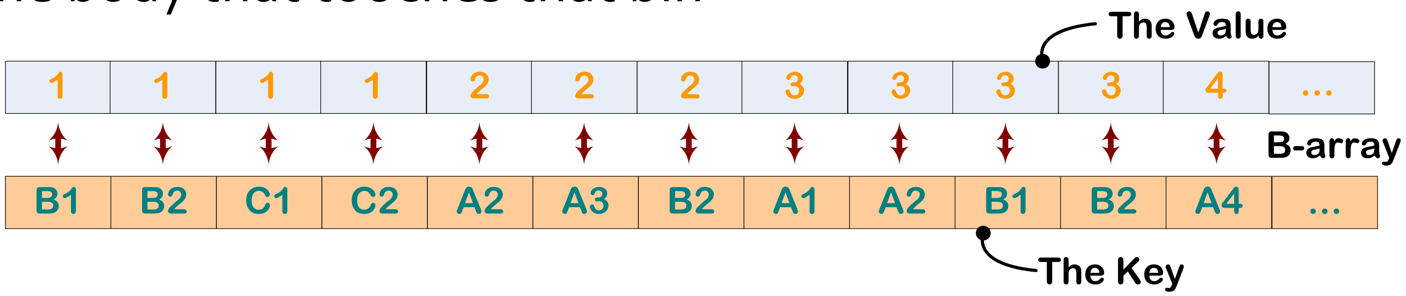
- Purpose: determine the number of entries M needed to store the indices of all the bins touched by each body in the problem



Stage 3: Determine bin-to-body association

- Stage executed in parallel on a per-body basis
- Allocate an array **B** of M pairs of integers.
- The key (first entry of the pair), is the bin index
- The value (second entry of pair) is the body that touches that bin

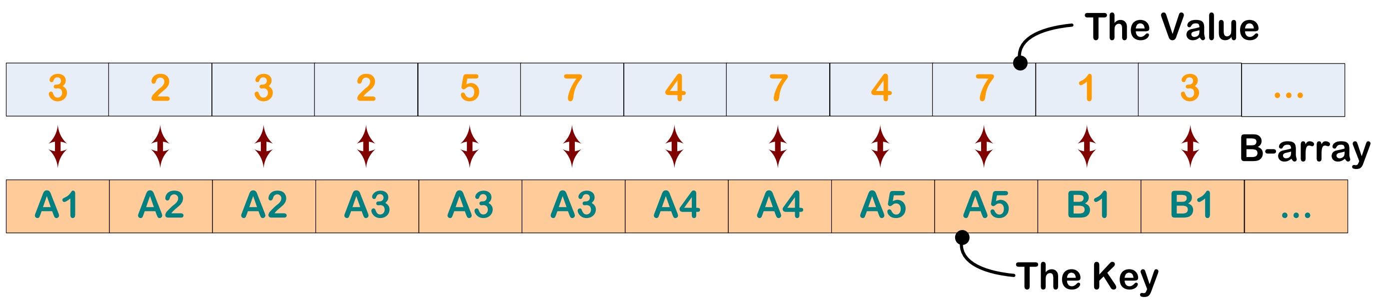


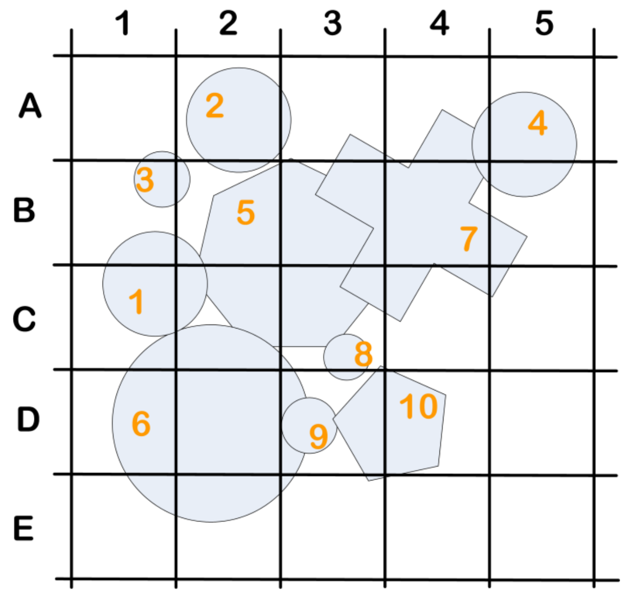


Stage 4: Radix Sort

- In parallel, run radix sort to order the **B** array according to key values

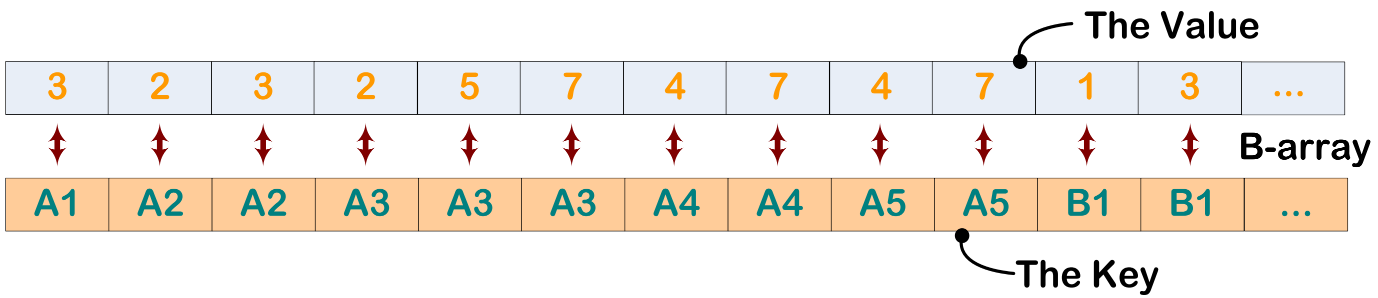
- Work load: O(N)
- Relies on **thrust** library

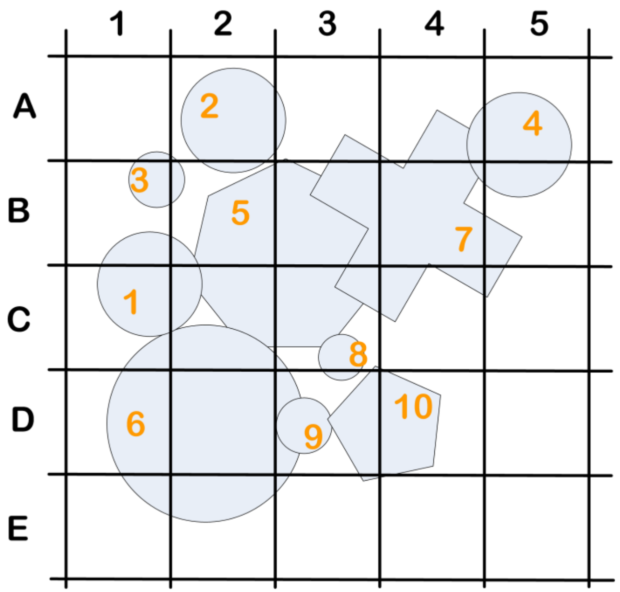




Stage 5: Find Bin Starting Index

- Host allocates on device an array of length N_b of pairs of unsigned integers
- Run in parallel, on a per bin basis:
  - Load in parallel in shared memory chunks of the **B** array and find the location where each bin starts
  - Store it in entry *k* of **C**, as the key associated with this pair
  - Key of bins with one or no bodies is set to maximum unsigned int value of 0xffffffff





Stage 6: Sort **C** for Pruning

- Do a parallel radix sort on the array **C** based on the key
- Purpose: move unused bins to the end of array
- Effort: $O(N_b)$

C-array

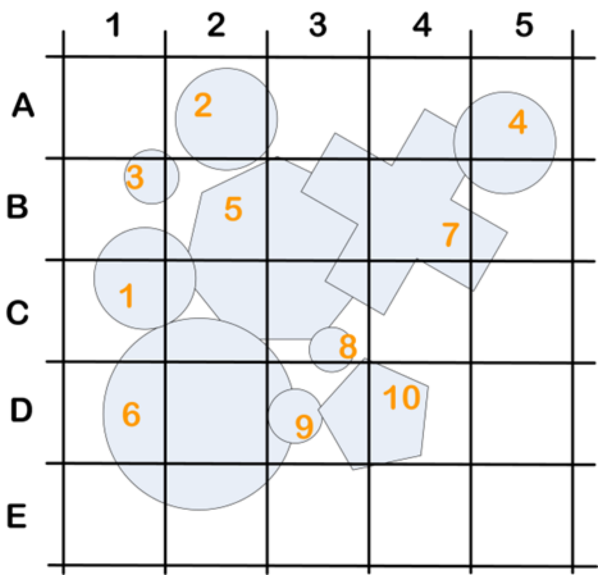
The Value	0	1	3	6	8	10	...
The Key	0xfff	2	3	2	2	2	...
	A1	A2	A3	A4	A5	B1	...

Becomes this

C-array

The Value	1	6	8	10	3	...	
The Key	2	2	2	2	3	...	

...



Stage 7: Investigate Collisions in each Bin

- Carried out in parallel, one thread per bin

B-array

0	1	2	3	4	5	6	7	8	9	10	11	
3	2	3	2	5	7	4	7	4	7	1	3	...
A1	A2	A2	A3	A3	A3	A4	A4	A5	A5	B1	B1	...

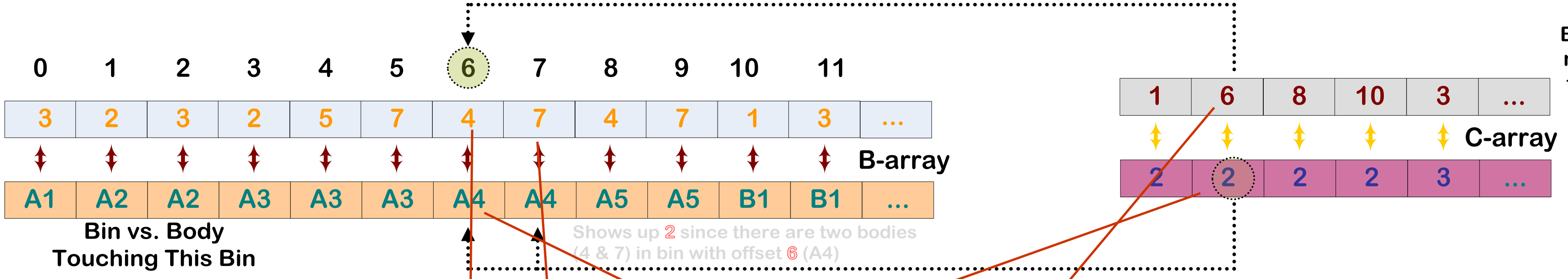
C-array

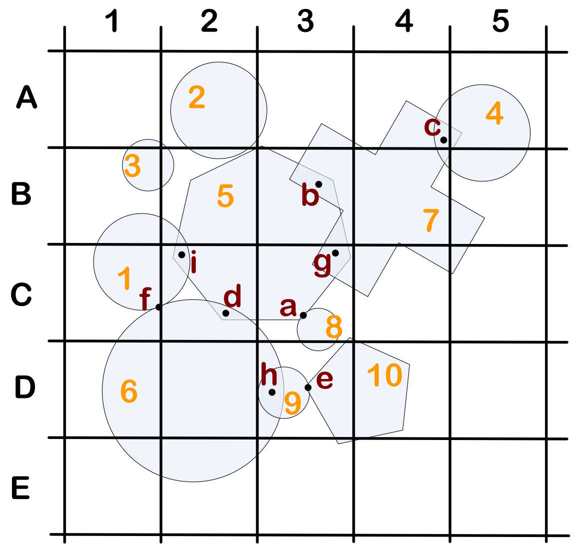
1	6	8	10	3	...
2	2	2	2	3	...

- To store information generated during this stage host allocates unsigned integer array **D** of length N_b
  - Array **D** stores the number of actual contacts occurring in each bin
  - **D** is in sync with (linked to) **C**, which in turn is in sync with (linked to) **B**
- Parallelism: one thread per bin
  - Thread *k* reads the pair key-value in entry *k* of array **C**
  - Thread *k* reads does rehearsal for brute force collision detection
  - Outcome: the number s of active collisions taking place in a bin
  - Value *s* stored in k^{th} entry of the **D** array

Stage 7: Details…

- Recall that how C is organized is a reflection of how B is organized





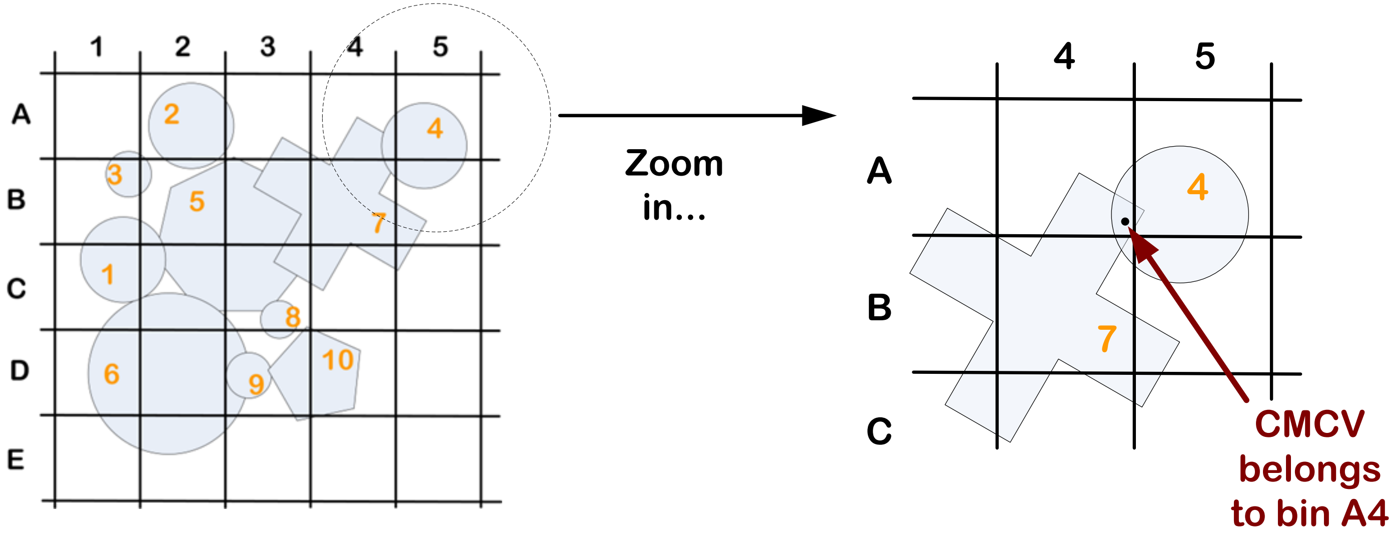
- The drill: thread 0 relies on info at **C**[0], thread 1 relies on info at **C**[1], etc.
- Let's see what thread 2 (goes with **C**[2]) does:
  - Read the first 2 bodies that start at offset 6 in **B**.
    - These bodies are 4 and 7, and as B indicates, they touch bin A4
    - Bodies 4 and 7 turn out to have 1 contact in A4, which means that entry 2 of D needs to reflect this

0	1	2	3	4	…
0	1	0	0	0	…
(A2)	(A4)	(A5)	(B1)	(A3)	…

D-array (Length: N_b)

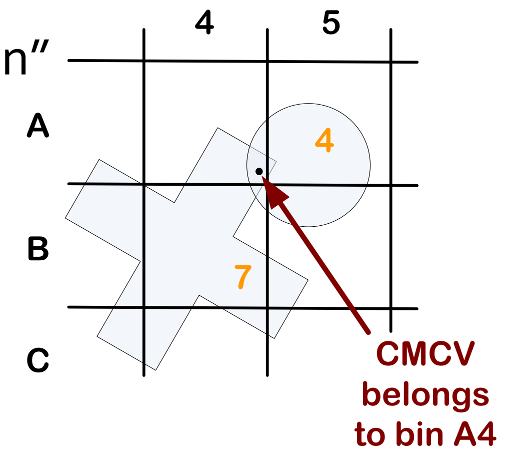
Stage 7: Details…

- Brute Force CD rehearsal
- Carried out to understand the memory requirements associated with collisions in each bin
- Finds out the total number of contacts owned by a bin
- Key question: which bin does a contact belong to?
- Answer: It belongs to bin containing the CM of the Contact Volume (CMCV)



Stage 7, Comments

- Two bodies can have multiple contacts, handled ok by the method
- Easy to define the CMCV for two spheres, two ellipsoids, and a couple of other simple geometries
  - In general finding CMCV might be tricky
    - Notice picture below, CM of 4 is in A5, CM of 7 is in B4 and CMCV is in A4
  - Finding the CMCV is the subject of the so called "narrow phase collision detection"
    - It'll be simple in our case since we are going to work with simple geometry primitives



Stage 8: Inclusive Prefix Scan

- Save to the side the number of contacts in the last bin (last entry of **D**) d_{last}
  - Last entry of **D** will get overwritten

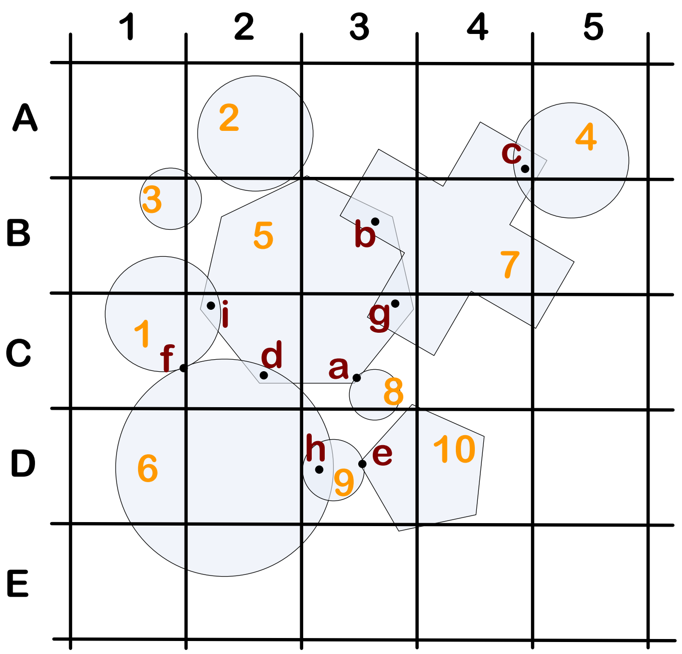
0	1	2	3	4	…
0	1	0	0	0	…
(A2)	(A4)	(A5)	(B1)	(A3)	…

D-array (Length: N_b)

- Run parallel exclusive prefix scan on **D**:

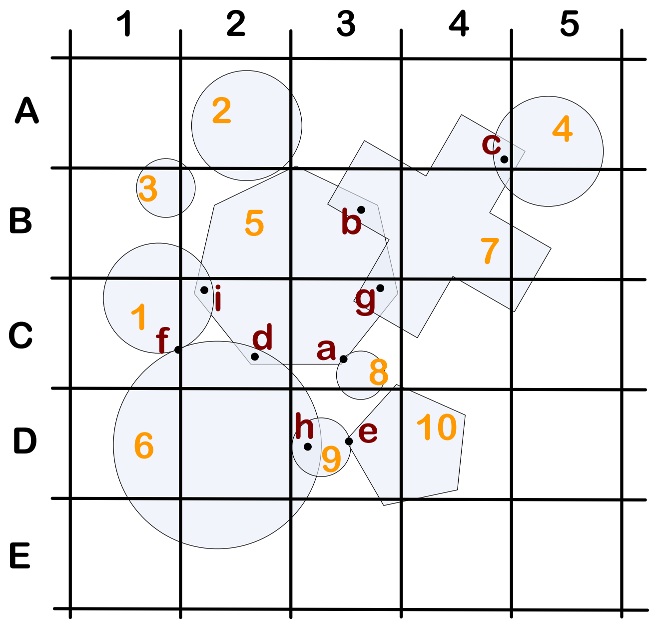
0	1	2	3	4	…
0	0	1	1	1	…
(A2)	(A4)	(A5)	(B1)	(A3)	…

D-array, after exclusive prefix scan



- Total number of actual collisions: $N_c = \mathbf{D}[N_b] + d_{last}$

Stage 9: Populate Array E



- From the host, allocate on the device memory for array **E**
  - Array **E** stores the required collision information: normal, two tangents, etc.
  - Number of entries in the array: N_c (see previous slide)
- In parallel, on a per bin basis (one thread/bin):
  - Populate the **E** array with required info
- Not discussed in greater detail, this is just like Stage 7, but now you have to generate actual collision info (stage 7 was the rehearsal)
- Thread for A4 will generate the info for contact "c"
- Thread for C2 will generate the info for "i" and "d"
- Etc.

Stage 9, details

- **B**, **C**, **D** required to populate array **E** with collision information

0	1	2	3	4	5	6	7	8	9	10	11	
3	2	3	2	5	7	4	7	4	7	1	3	…
A1	A2	A2	A3	A3	A3	A4	A4	A5	A5	B1	B1	…

B-array

Bin vs. Body Touching This Bin

Shows up 2 since there are two bodies (4 & 7) in bin with offset 6 (A4)

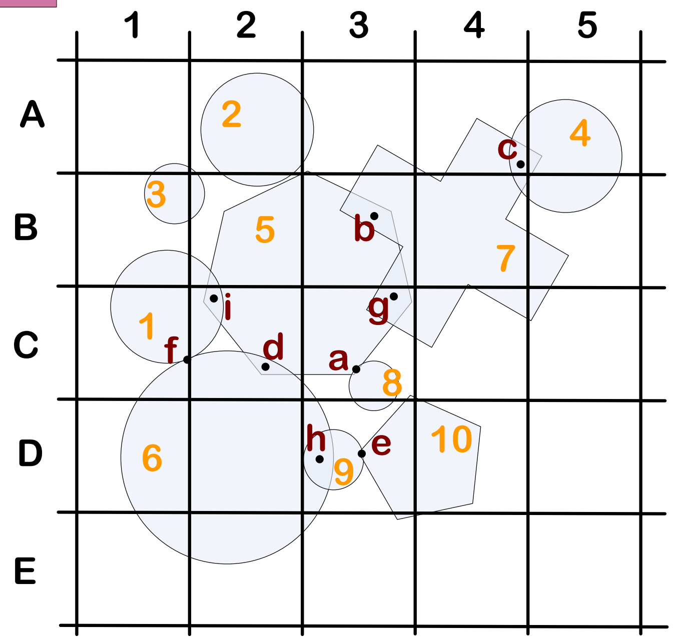
Bin offset in B and number of bodies touching that bin

1	6	8	10	3	…
2	2	2	2	3	…

C-array

0	1	2	3	4	…
0	0	1	1	1	…
(A2)	(A4)	(A5)	(B1)	(A3)	…

D-array, after exclusive prefix scan

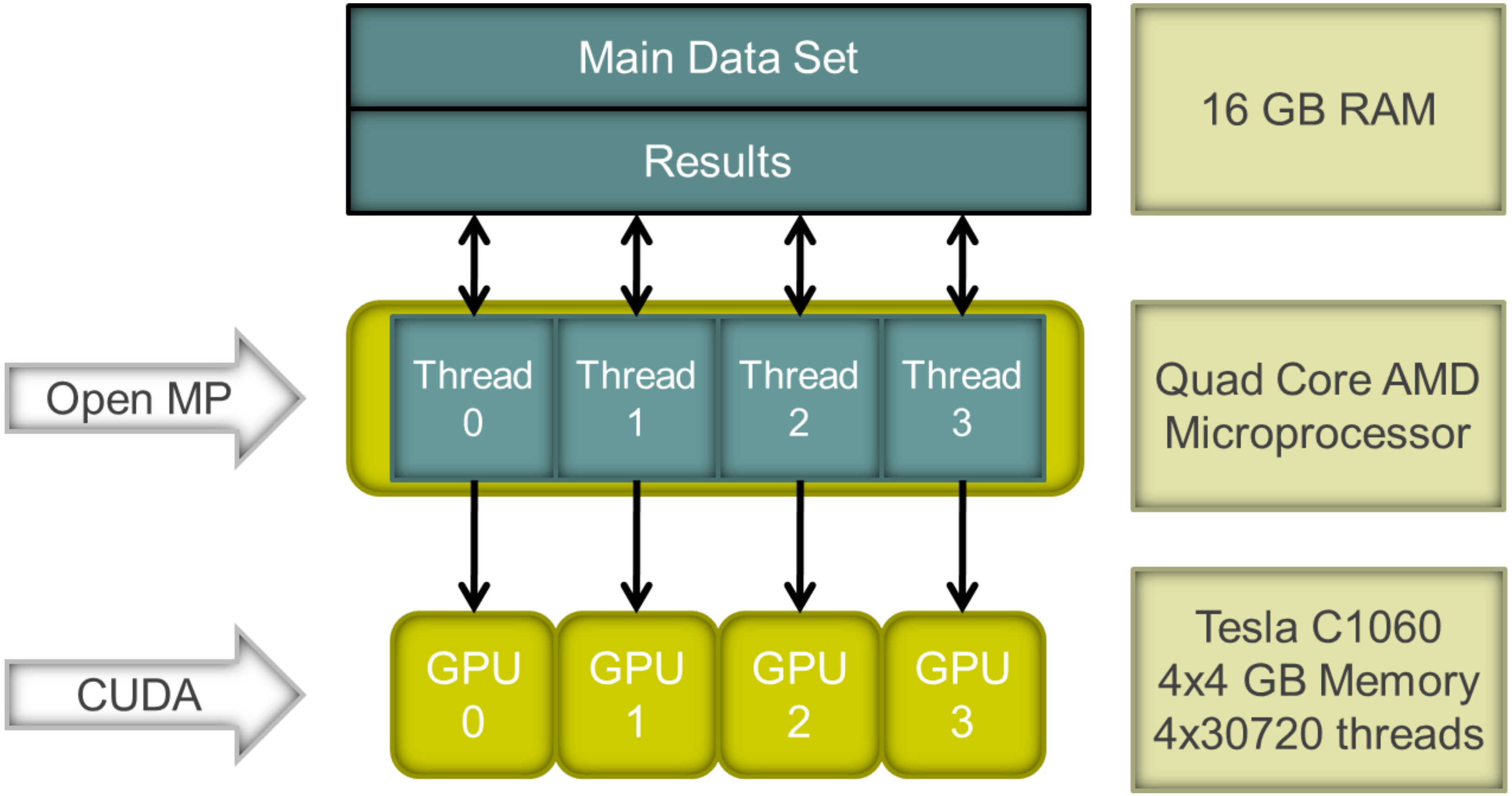


- **C** and **B** are needed to compute the collision information
- **D** needed to understand where collision information is stored in **E**

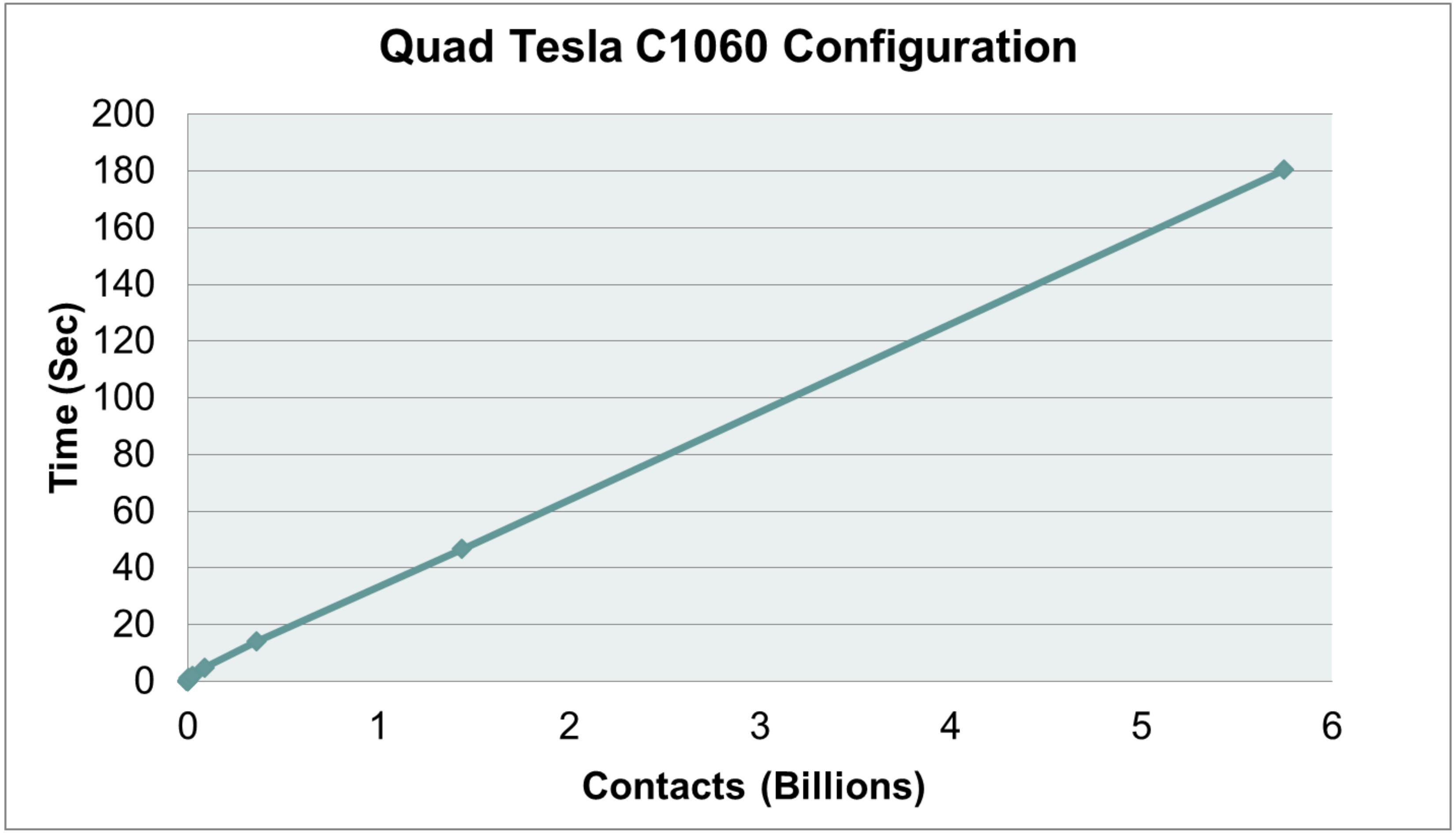
Multiple-GPU Collision Detection

- Processor: AMD Phenom II X4 940 Black
- Memory: 16GB DDR2
- Graphics: 4x NVIDIA Tesla C1060
- Power supply 1: 1000W
- Power supply 2: 750W

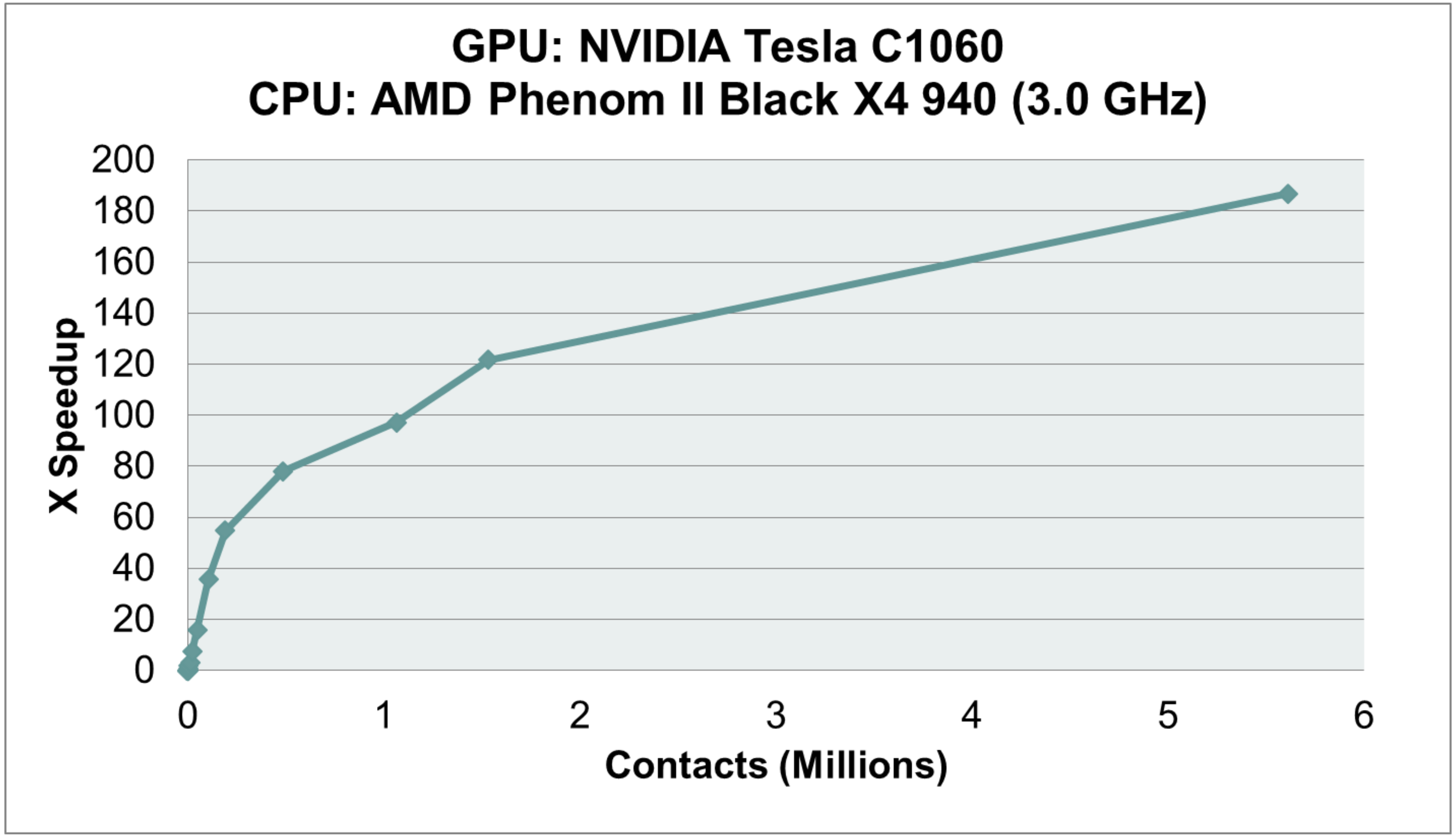
Software/Hardware Setup



Spheres – Contacts vs. Time



Speedup - GPU vs. CPU (Bullet library)
[results reported are for spheres]



Domain decomposition
&
Inter-domain data exchange

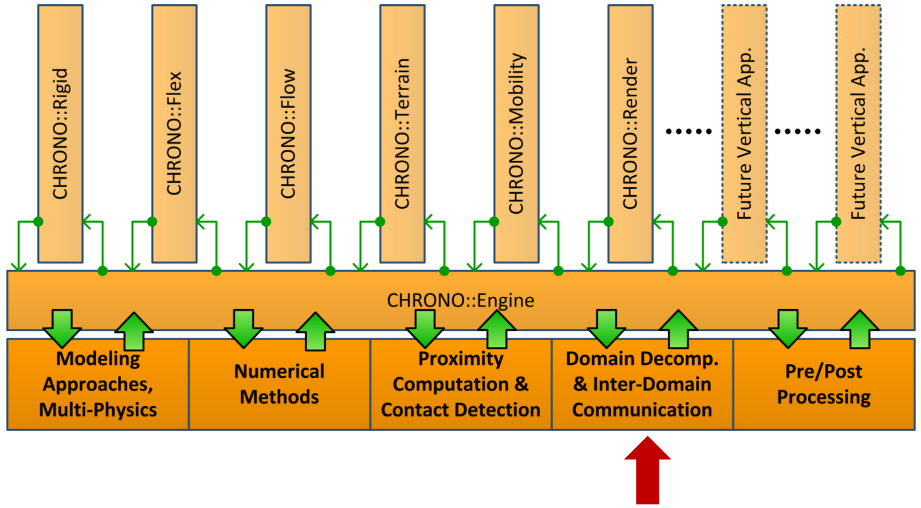
Advanced modeling techniques

Algorithmic (applied math) support

Proximity computation

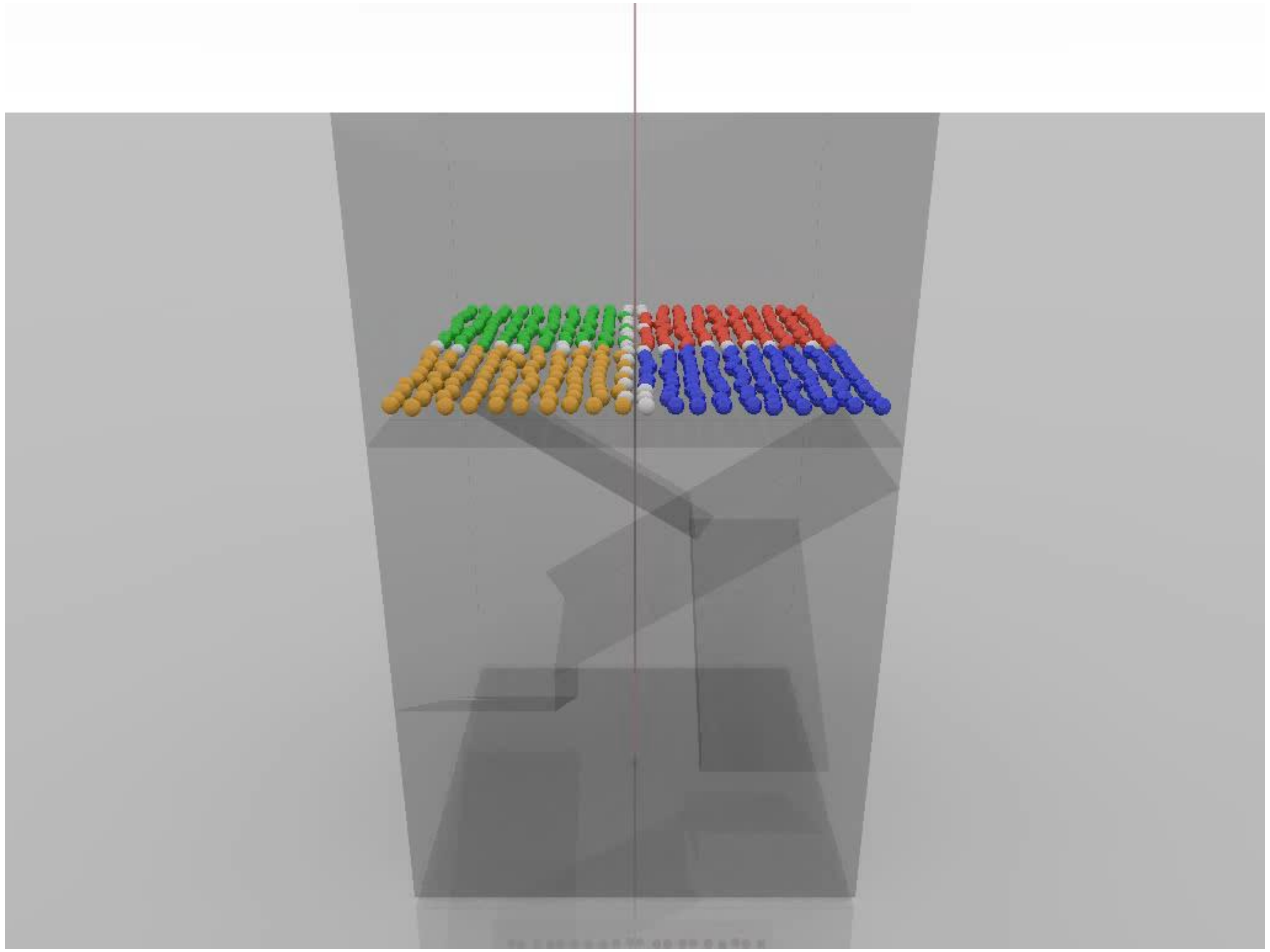
Domain decomposition & Inter-domain data exchange

Post-processing (visualization)



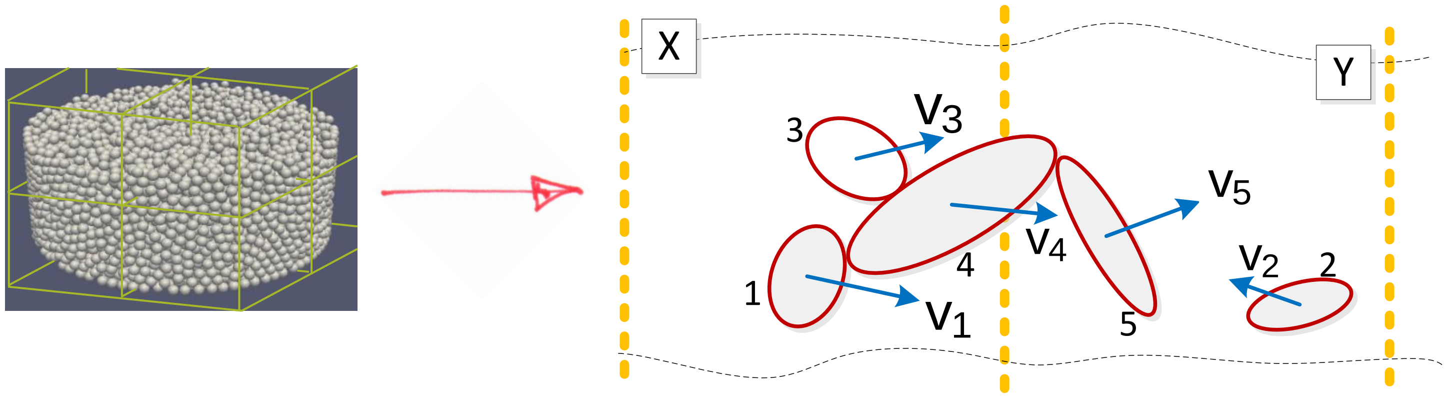
Juggling World Record:
64 People Juggling (of all places) in Madison, Wisconsin

Computation Using Multiple CPUs

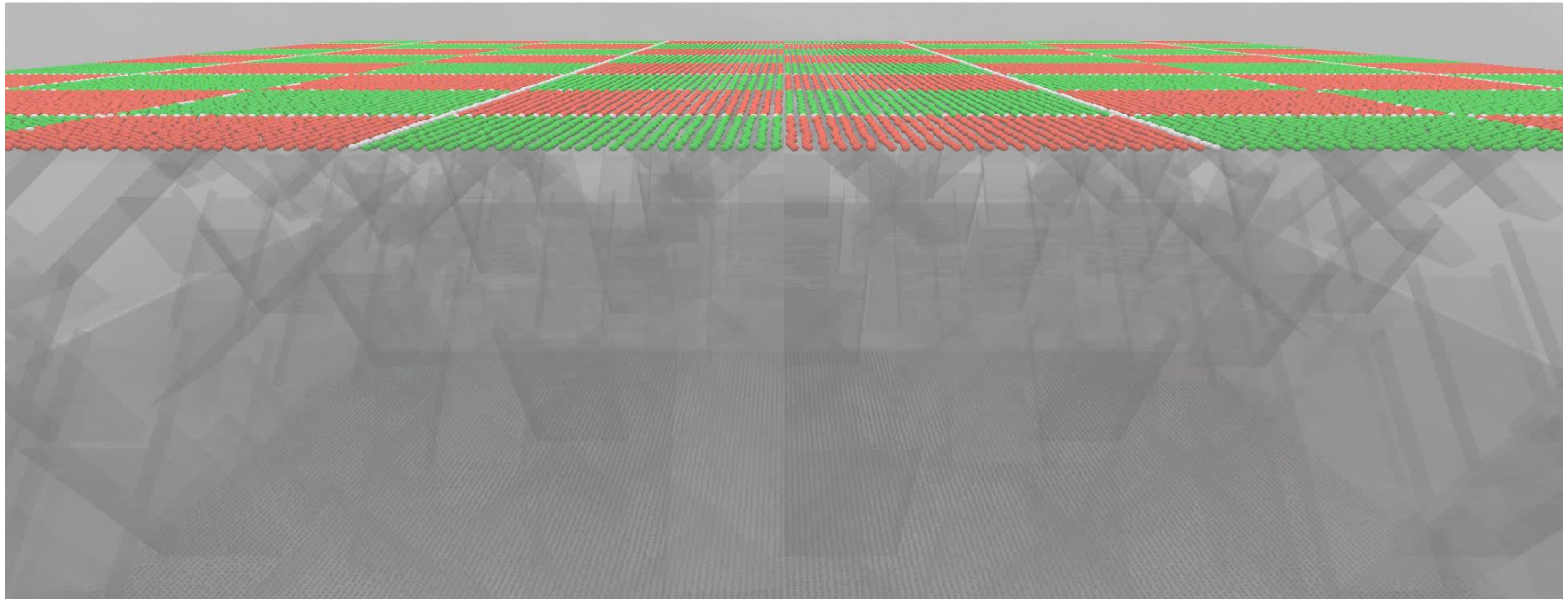


CHRONO:
Domain decomposition & Inter-domain data exchange

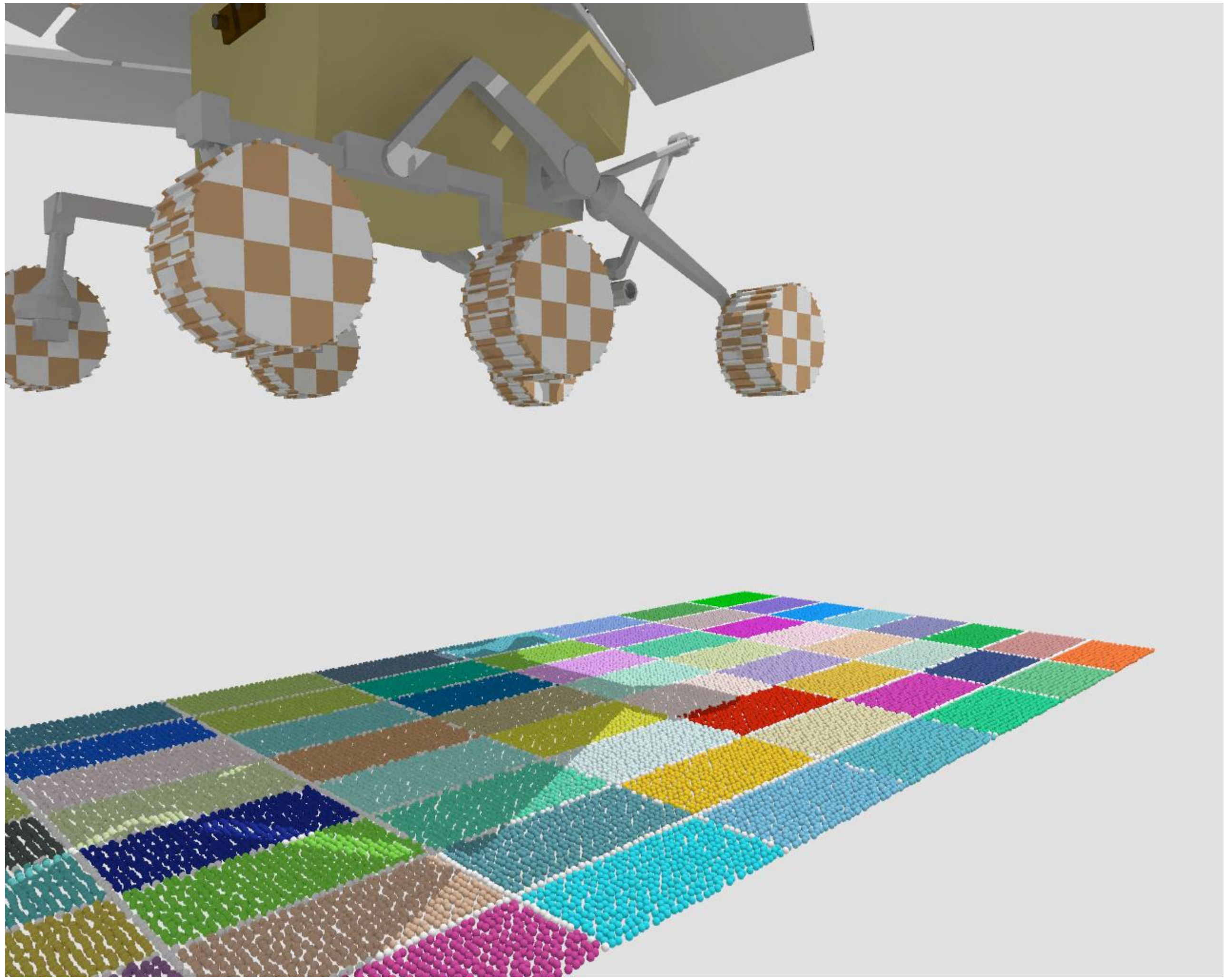
- Divide simulation into chunks and have multiple CPUs/GPUs exchange data during simulation, as needed
- Elements leave one subdomain to move to a different one in transparent fashion
- Key issues:
  - Dynamic load balancing
  - Establish a dynamic data exchange protocol (DDEP) between sub-domains



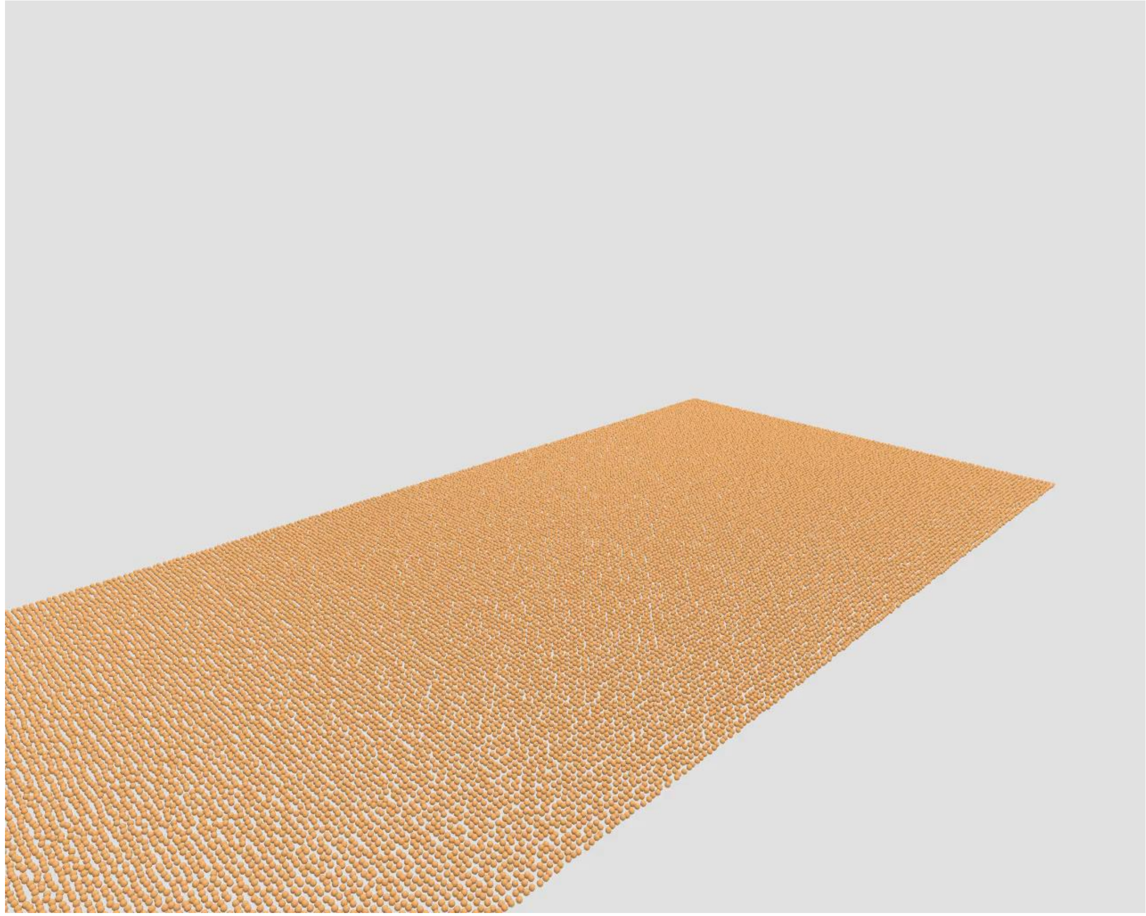
0.5 Million Bodies on 64 Cores
[Penalty Approach, MPI-based]



Computation Using Multiple CPUs



Rover Footprint, Multi-Domain Computation



Pre/Post-processing (visualization)

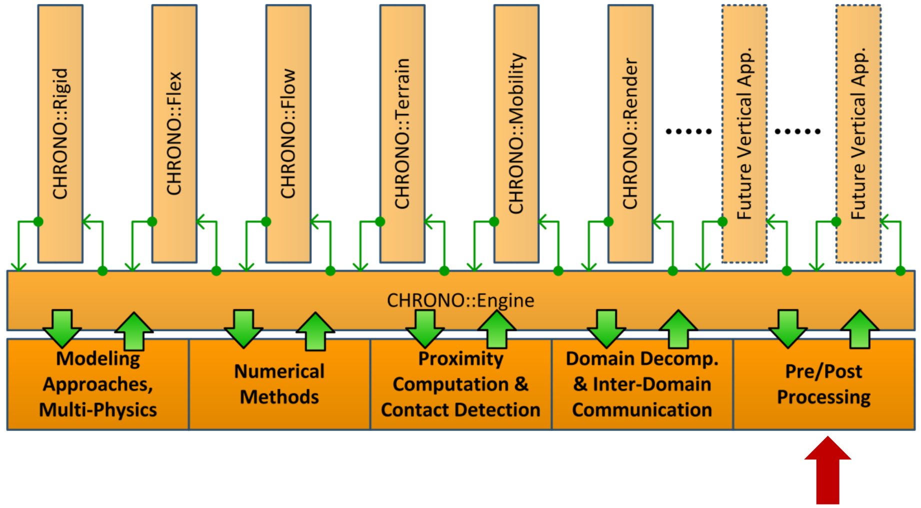
Advanced modeling techniques

Algorithmic (applied math) support

Proximity computation

Domain decomposition & Inter-domain data exchange

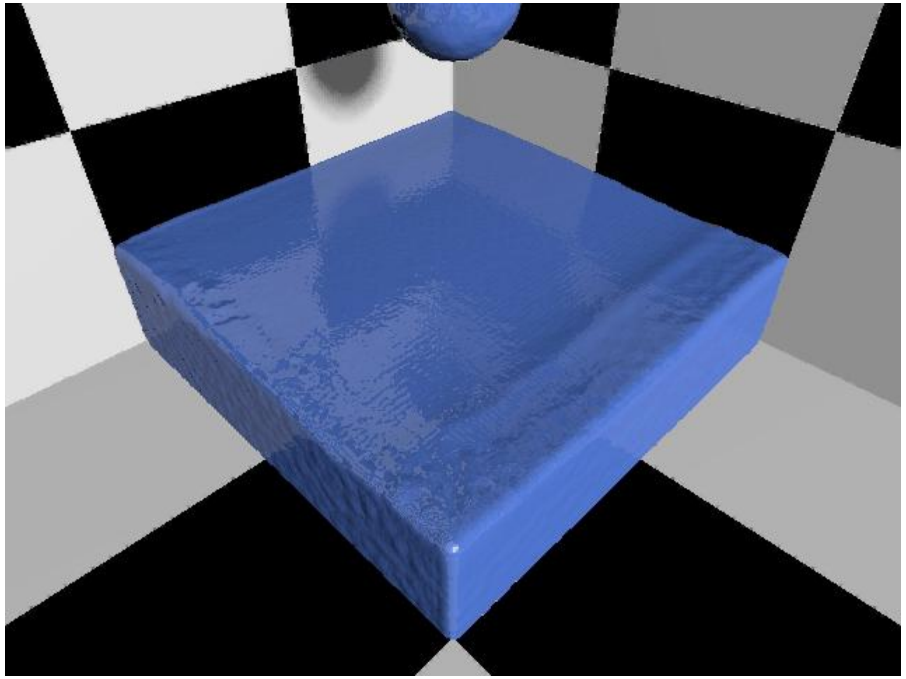
Post-processing (visualization)

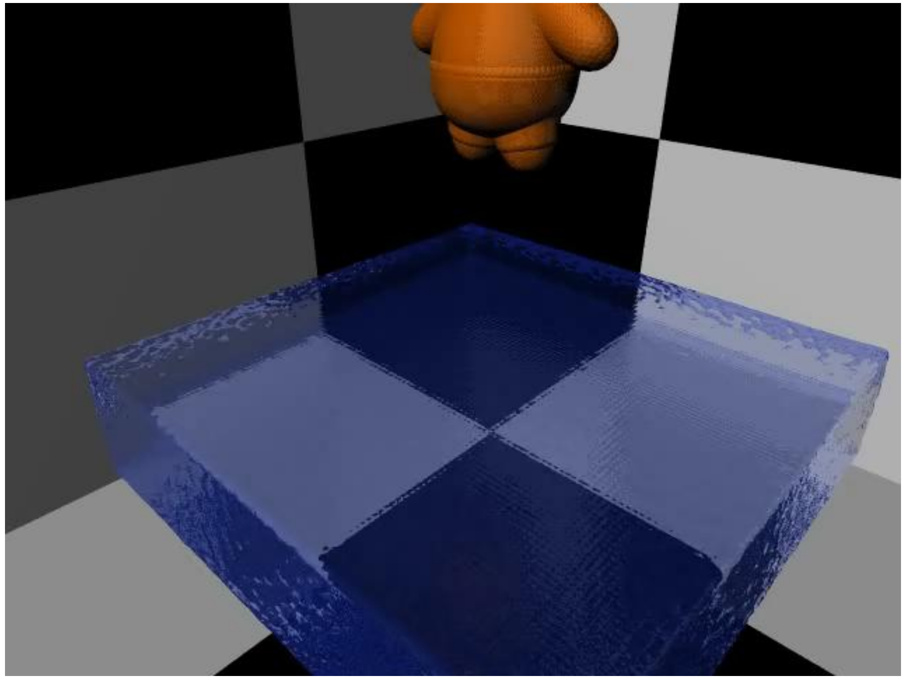


CHRONO: Visualization and Post-Processing

- Rendering very complex scenes with more than one million components
- Rendering takes longer than simulating
- Pursuing a rendering pipeline that leverages parallel computing

Fluid Dynamics and Fluid-Solid Interaction





Rendering Pipeline: Problem Statement

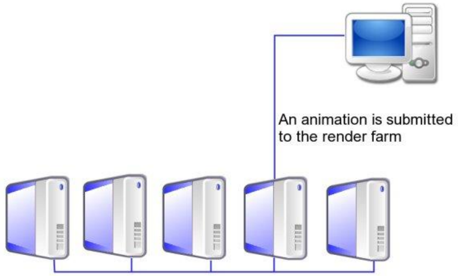
- Render big data: efficiently and beautifully
- Have the flexibility to render anything
- Make the rendering process streamlined and simple
  - Provide rendering as a service

Rendering Pipeline: From Data to Movie

- Example: Tire-Terrain Simulation
  - Data
    - Sequence of Height Maps
    - Render Settings File
  - Cluster
    - Submit data and settings to cluster remotely
    - Schedule jobs and render
  - Movie
    - Returned upon completion

DATA

CLUSTER



MOVIE

UNCLASSIFIED: Distribution Statement A. Approved for public release. #24389

Tire Rolling on Deformable Terrain

Tire Rolling on Deformable Terrain

Rendering Pipeline Uses Pixar's Renderman (PRMan)

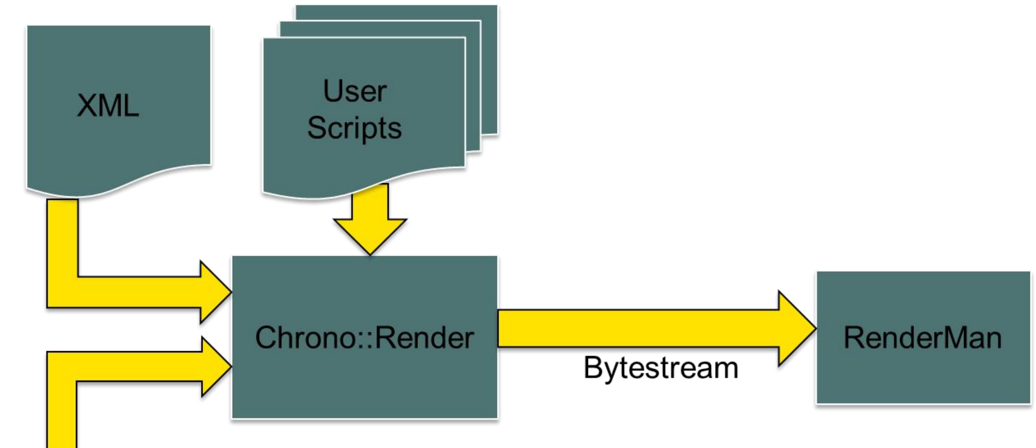
- PRMan: Engineered to be fast, efficient, and configurable for complex scene rendering

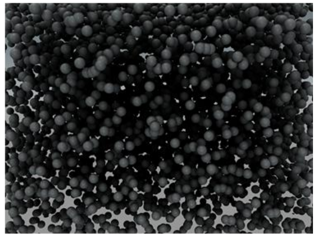
- Pixar's PRMan: industry's rendering standard
  - Lab supercomputer can run up to 320 instances of PRMan

- Open source alternatives:
  - Aqsis
  - JrMan
  - Pixie

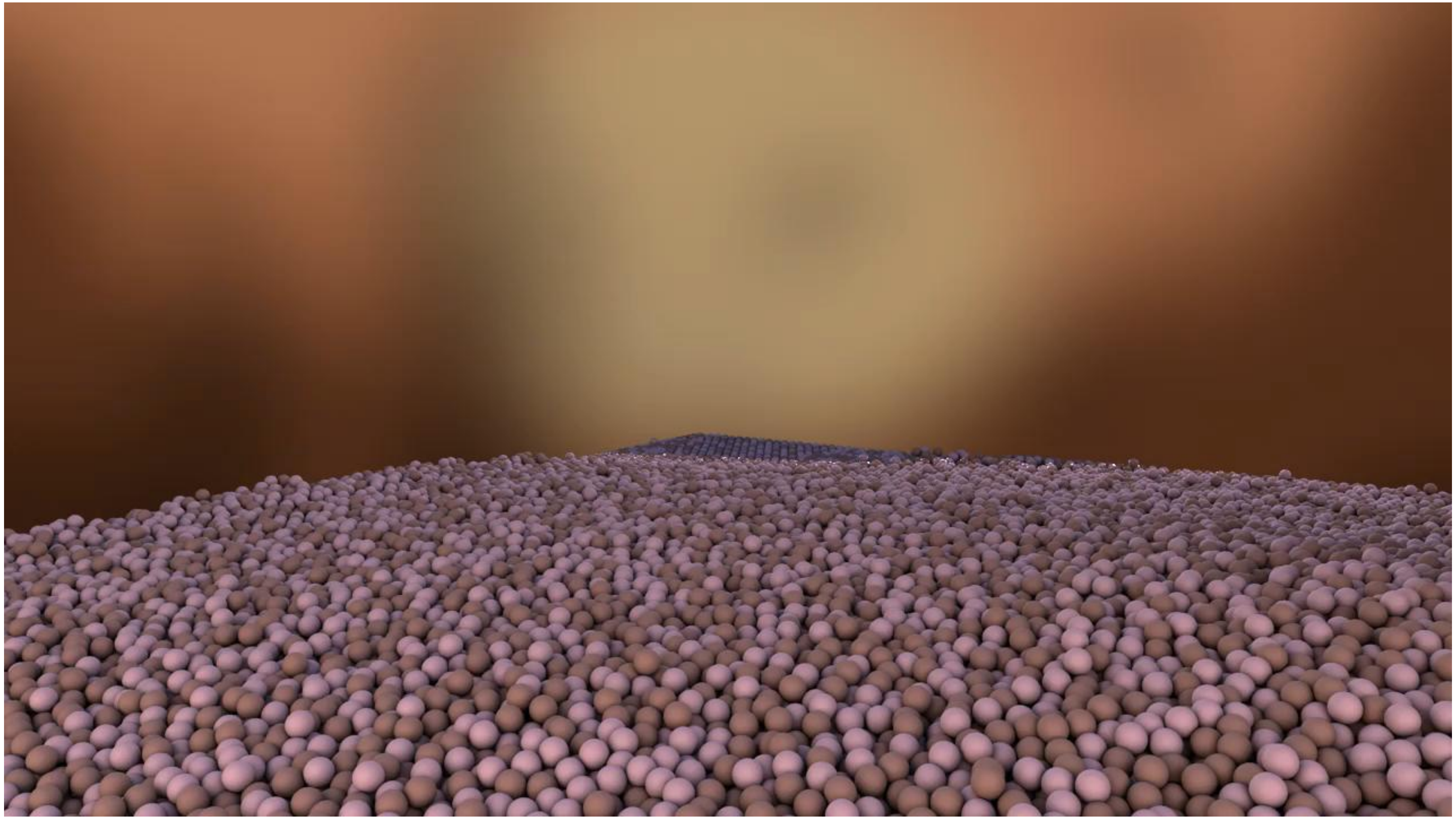
Chrono Rendering Pipeline:

- RenderMan requires a lot of work to configure and optimize correctly
  - Not aimed at science and engineering communities
- Chrono::Render tailored to science and engineering
- Chrono::Render - What is it?
  - C++ binaries, simple Python scripting interface, and succinct XML specification

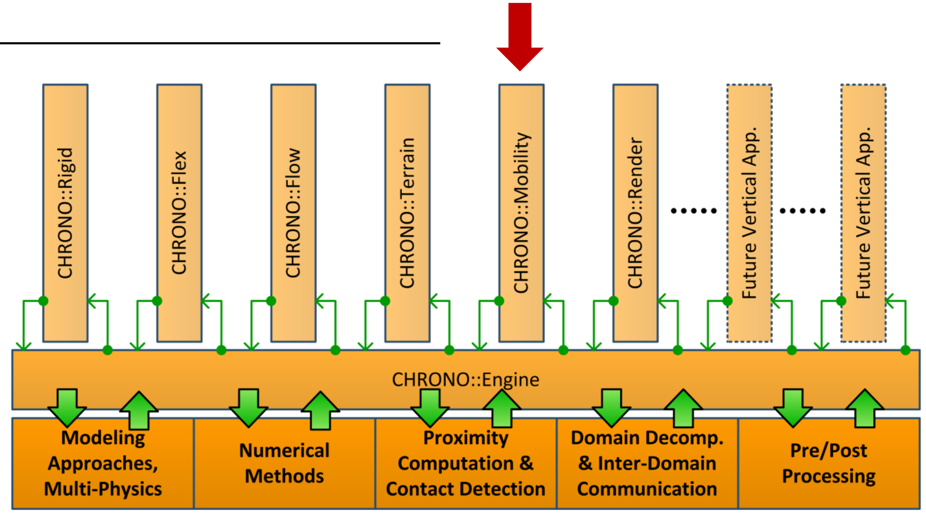




Light Robot Operating on Discrete Terrain



Chrono::Mobility



Terramechanics Modeling Methodologies

1. Empirical methods
   - WES numerics, NATO Reference Mobility Model (NRMM)

2. Semi-analytical
   - Bekker-Reece vertical pressure/sinkage relation
   - Janosi-Hanamoto slip/shear relationship
   - Wong/Reece plastic equilibrium approach

3. Physics-based
   - Finite Element Analysis
   - Particle/Discrete Element methods (DEM, DVI)
   - Meshless/Lagrangian Methods (SPH, MPM, etc.)

Terramechanics for Vehicle Mobility, Remarks

- Empirical methods have limited predictive attributes for general purpose vehicle mobility
- Semi-Analytical methods have been applied to mobility studies with some success
  - See: Trease, Holtz, Azimi, Schmid, Harnisch, Slattengren
  - Limitations due to some (but not necessarily all) of the following assumptions:
    1. Tire geometry is 2-D, circular in shape
    2. Wheel moves forward at a constant velocity and spin rate
    3. Wheel moves parallel to flat ground
    4. Soil is homogenous, perfectly plastic medium
- FEA or DEM are accurate, but computationally expensive
  - Madsen/Heyn/Negrut/Lamb (demonstrated shortly)

Chrono::Mobility – Goals

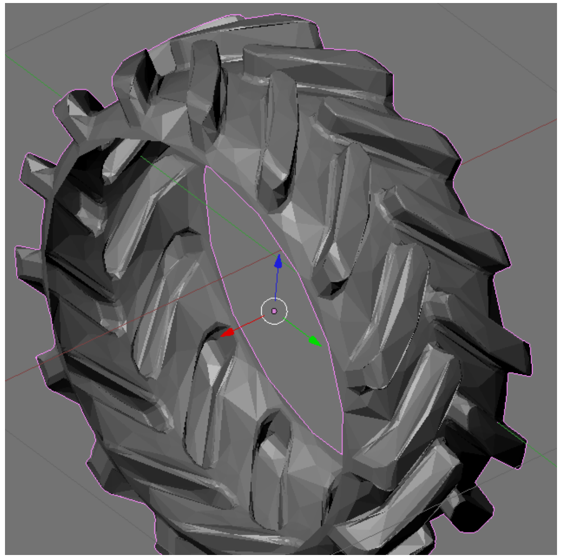
- Develop general purpose simulation capability for analysis of wheeled/tracked vehicle mobility on deformable terrain

a) Handle 3-D tire/track geometry to accurately estimate contact patch size and shape

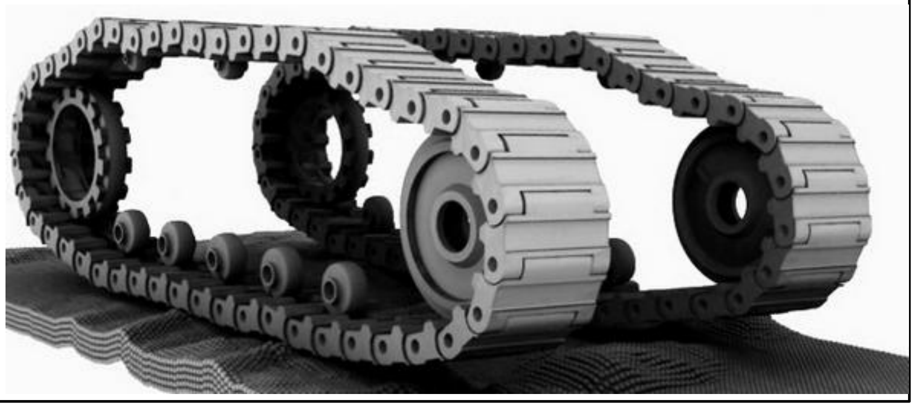
b) Handle general 3-D terrain geometry to allow for realistic mobility scenarios

c) Represent the terrain in a way that considers soil stress state and loading history in a volumetric sense, depending on soil type
   - Cohesive soils – compaction
   - Dry granular soils – shear failure and flow
   - Brittle soils – fracture, shear failure and flow

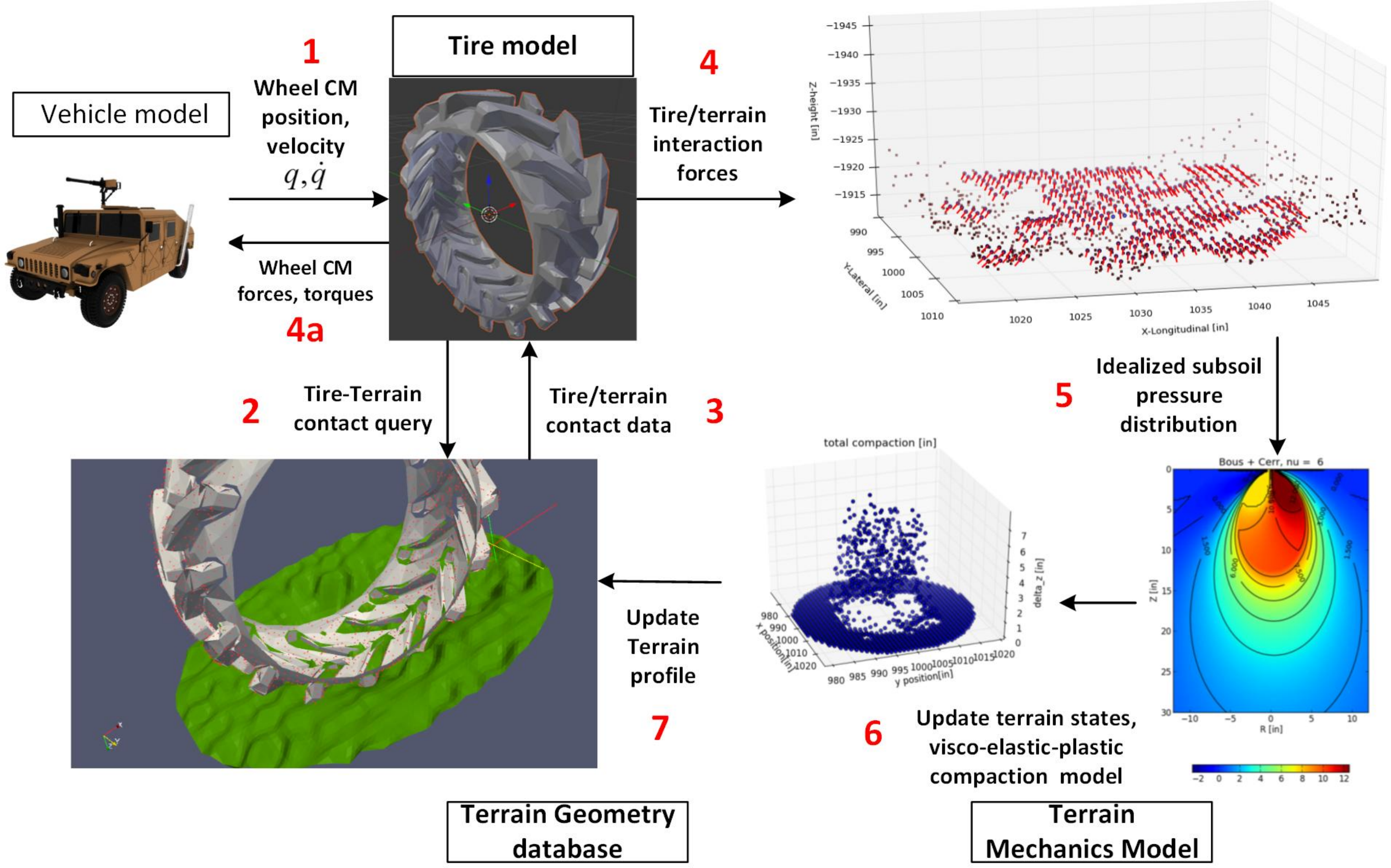
Traction Element Geometry Representation



- 3-D geometry description for tire/terrain collision query
  - Discretized at a resolution to capture tread/lug geometry
- APIs modularized so that terrain database accepts generalized traction geometry when queried
- Directly use Wavefront (.obj) solid models (top)
  - Captures complicated tread/lug geometry
- Can also represent vehicle hull geometry

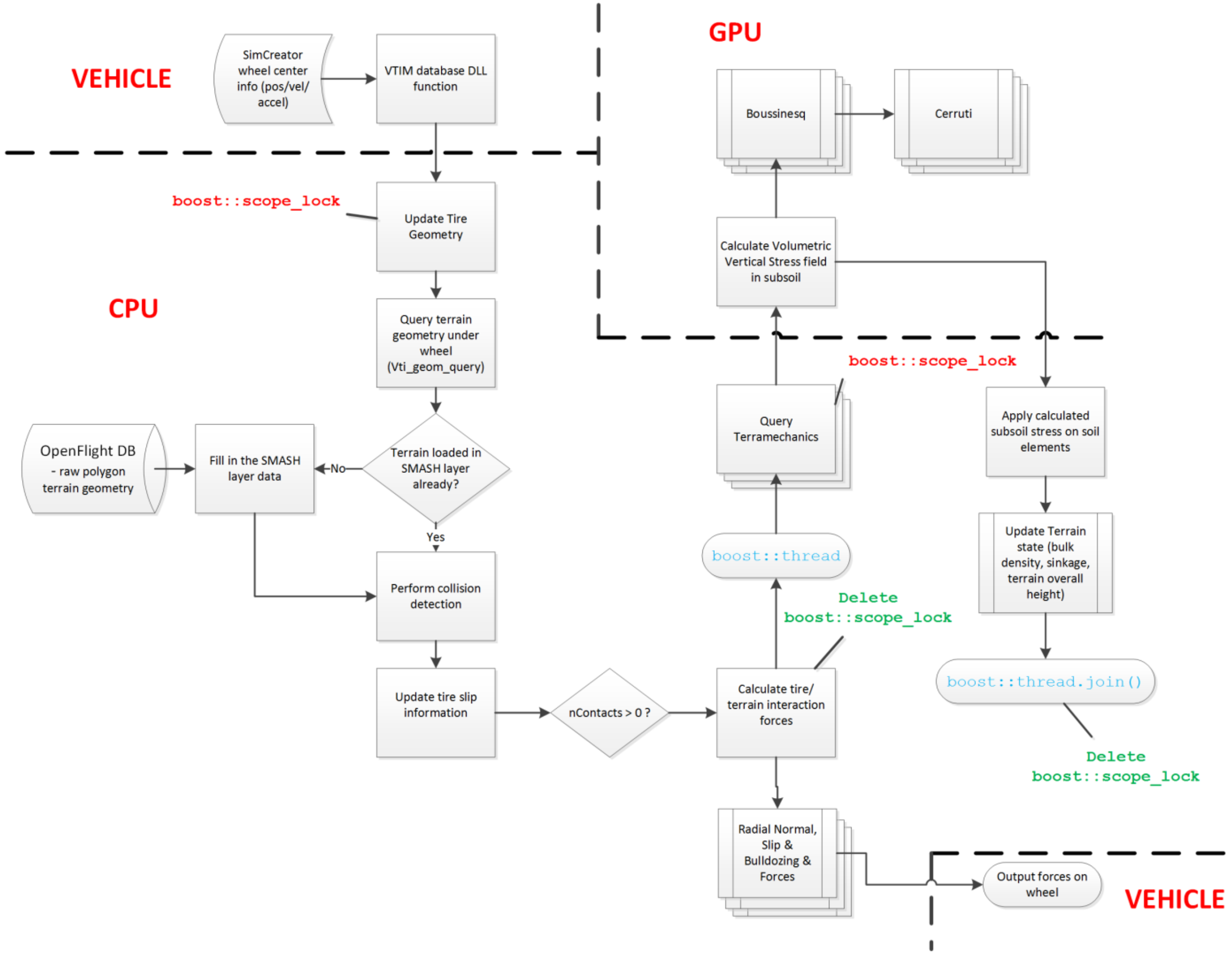


Simulation Framework



Simulation Framework: Implementation Details

- Fast, asynchronous execution
  - Not necessary for vehicle model to wait for query to complete
- Pass STI Force/Moment to vehicle
- Leverages both multi-core and GPU parallelism



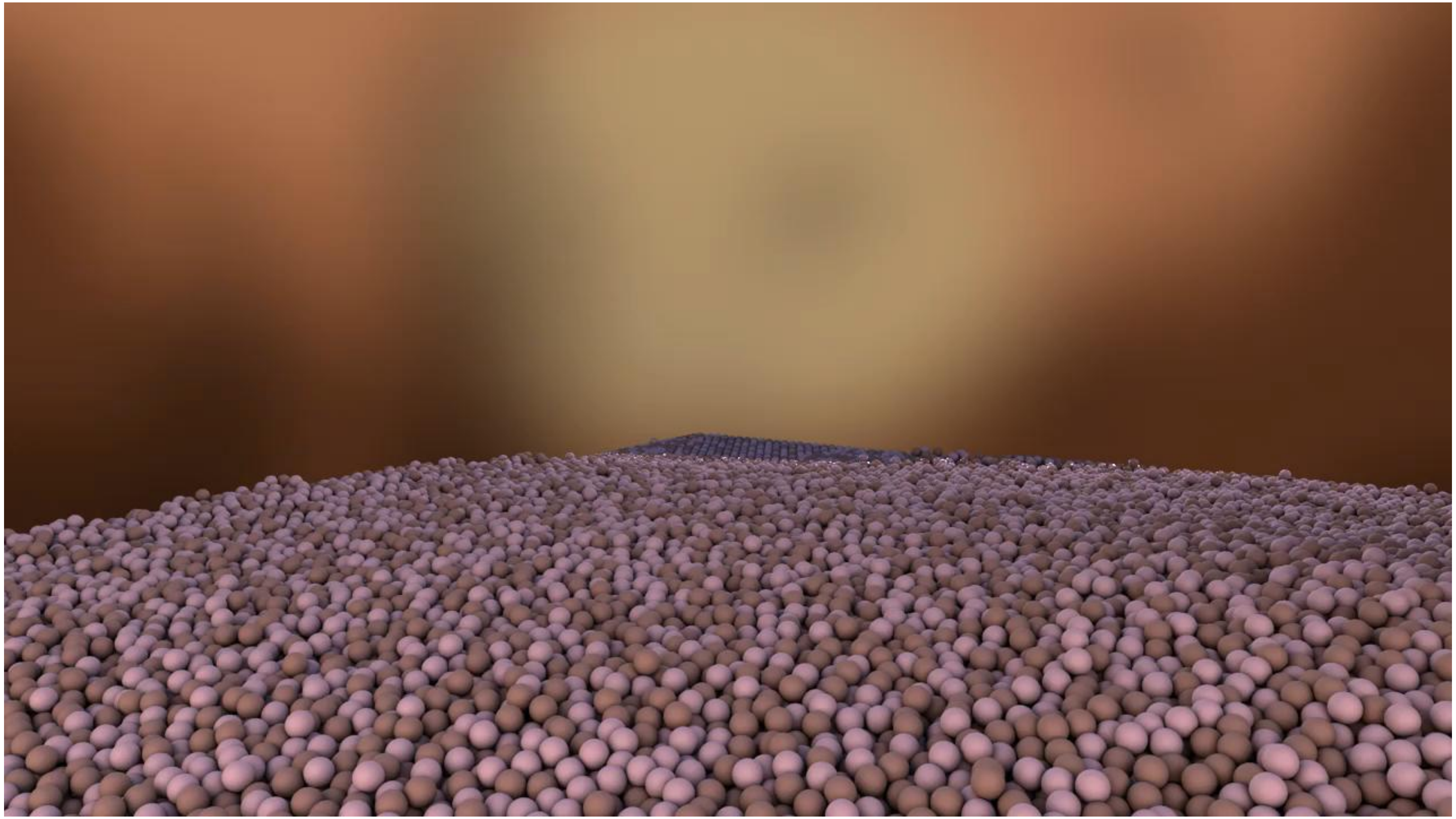
Chrono::Mobility, Today

- Multibody dynamics vehicle model entirely in **Chrono**
  - Generalized 3-D tire/track and terrain geometry representations
  - Visco-elastic-plastic soil model captures soil compaction due to vehicle loads

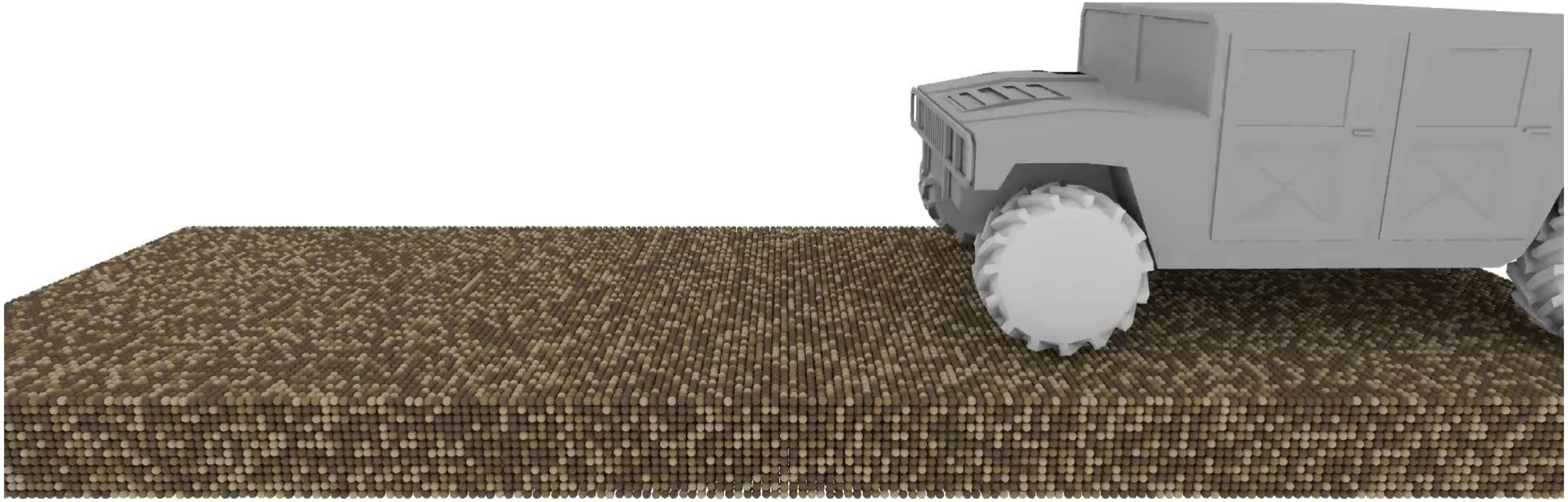
- Leverages other members of the **Chrono** family
  - Based on Standard Tire Interface (STI) & Vehicle Terrain Interface VTI

- Discrete terrain simulation carried out in the same framework

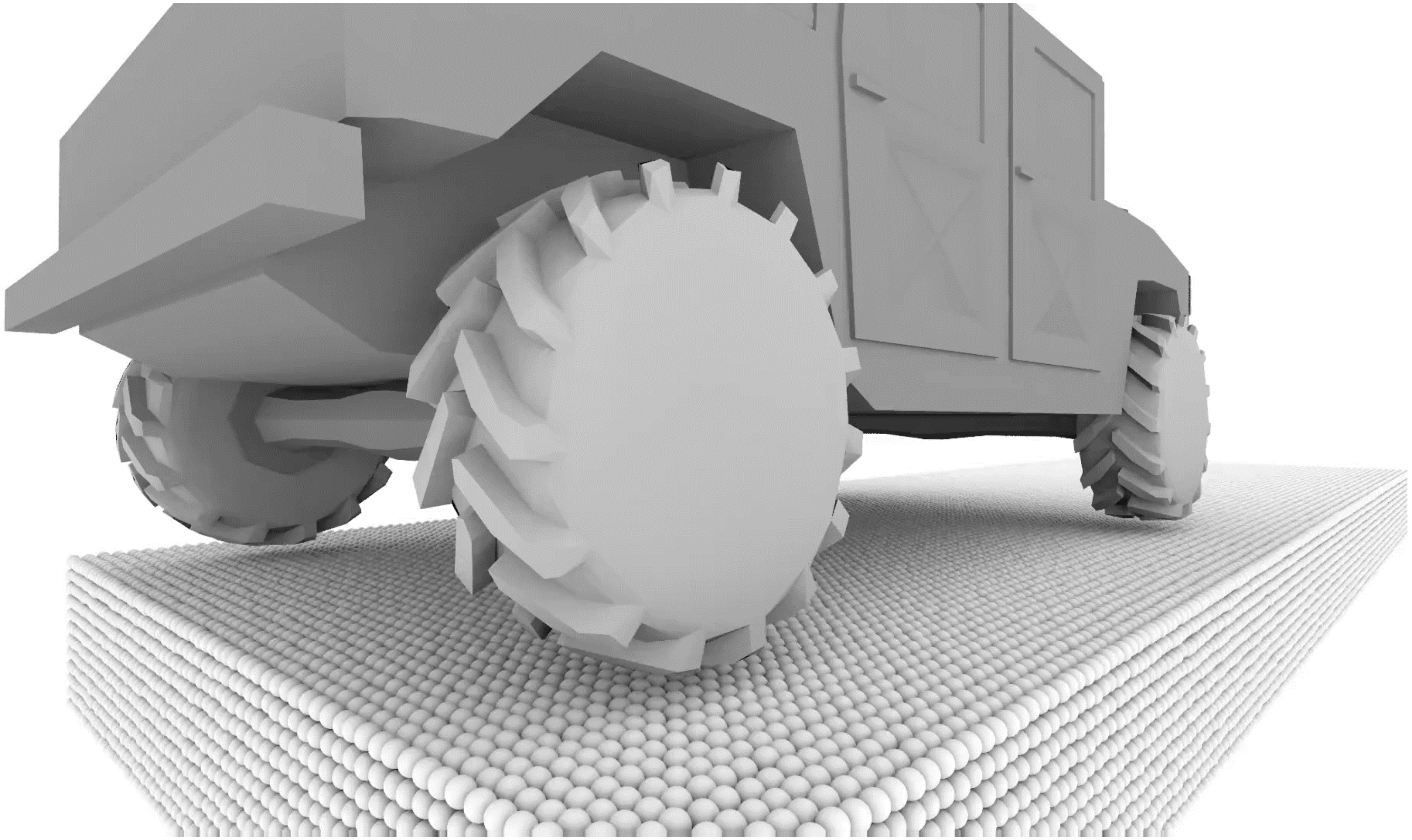
Light Robot Operating on Discrete Terrain



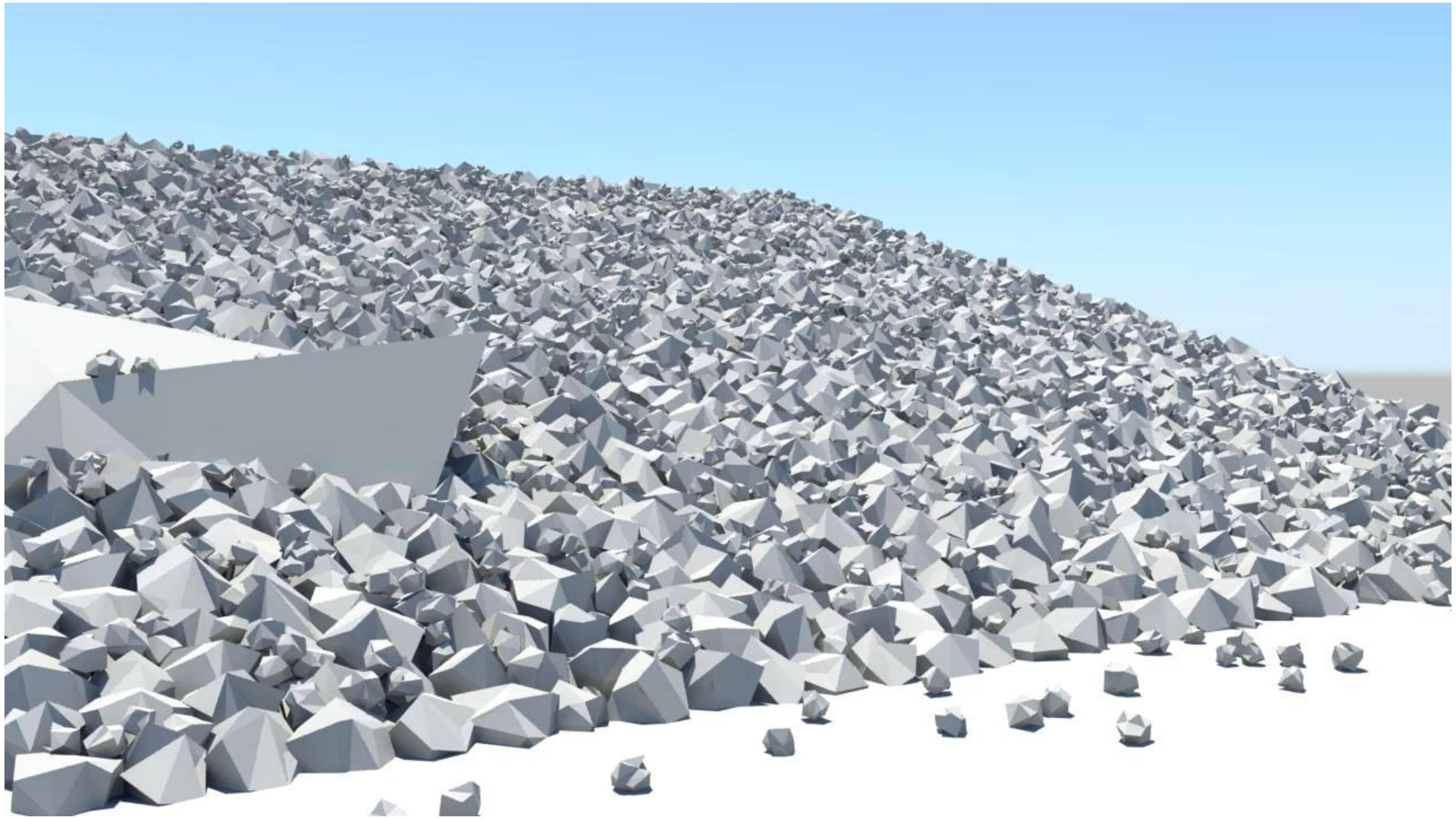
Mobility on Granular Terrain w/ Cohesion



Mobility on Granular Terrain w/ Cohesion: Close-Up

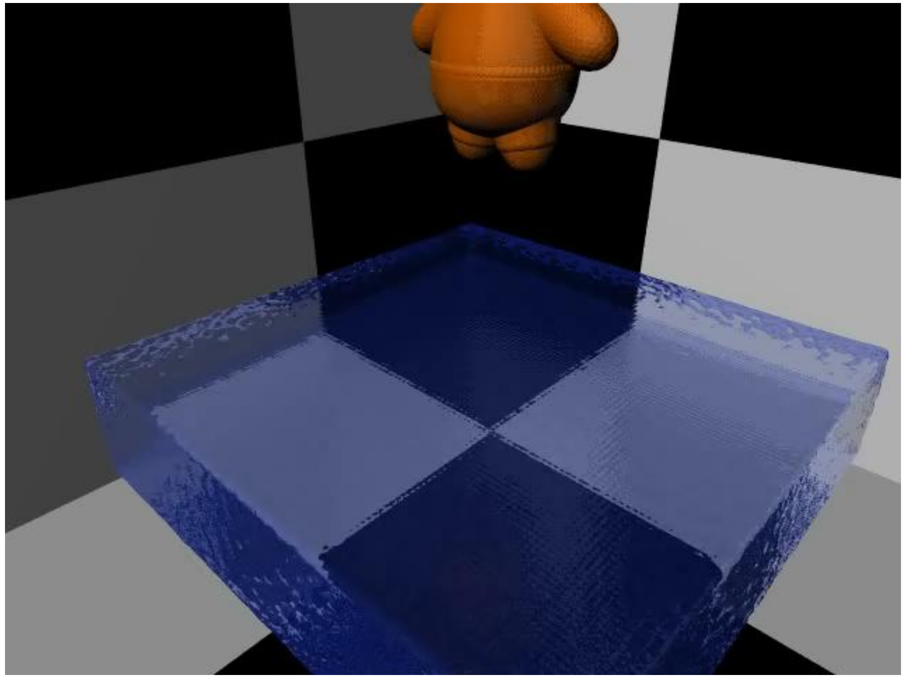


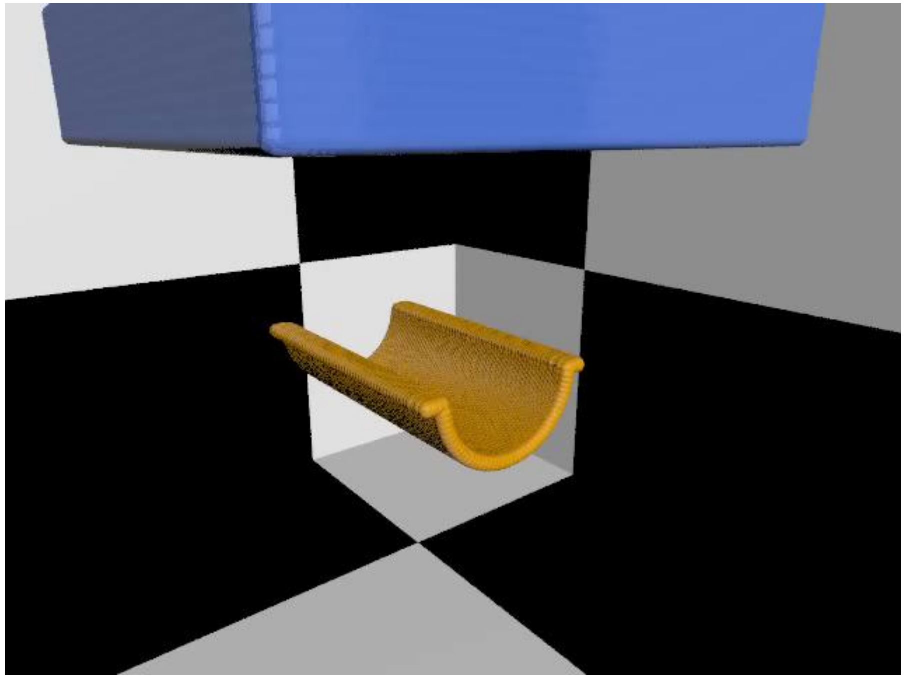
What Can You Do With a Validated Predictive Tool?



Fording, First Cut

Fording, Future Plans: Coupled Fluid-Structure Interaction





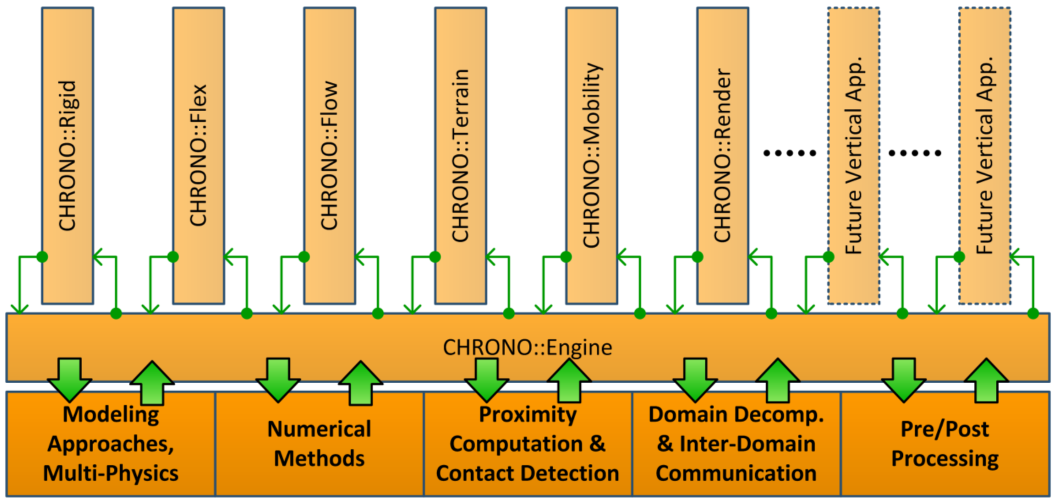
9.3 years of GPU time for simulating Fluid-Solid Interaction problems

Departing Thoughts

Chrono – The Long View

- **Chrono**-effort focused on four thrusts:

  1. Validation – useful
  2. Pre/Post – friendly
  3. New features – versatile
  4. Leverage advanced computing – fast



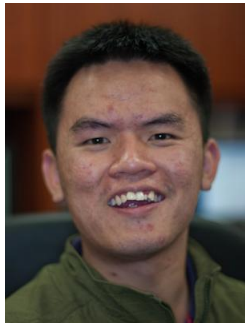
Wisconsin Applied Computing Center

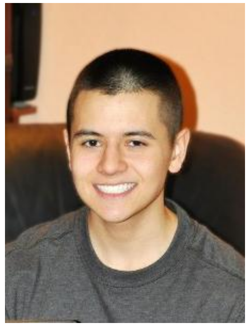
- Our group, Computational Dynamics, has 12 members
  - Three Faculty/Researchers…

- Nine Graduate and Undergraduate Students…








Closing Remarks

- We are focused on physics-based simulation

- Vision:
  - Solve real-world problems (pursue relevant questions)
  - Put computers and good ideas to work
  - Build upon partnerships and collaborations
  - Make outcomes of our work available to users: release early, release often (BSD-3 license)

- Approaching physics-based simulation in a holistic fashion through `Chrono`
  - Modeling + numerical solution + visualization
  - Rely on emerging hardware for fast simulation

Thank You.

negrut@wisc.edu

Simulation Based Engineering Lab

Wisconsin Applied Computing Center

PPT presentation & animations will be available on-line for download.